

UNIT 1: ABSOLUTE VALUE AND RADICALS

Note-Taking Supplement

Student Package

Student's Name: _____

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Lesson 1
Real Numbers and Absolute Value

1.1 Real Numbers

Classifications of numbers by type.

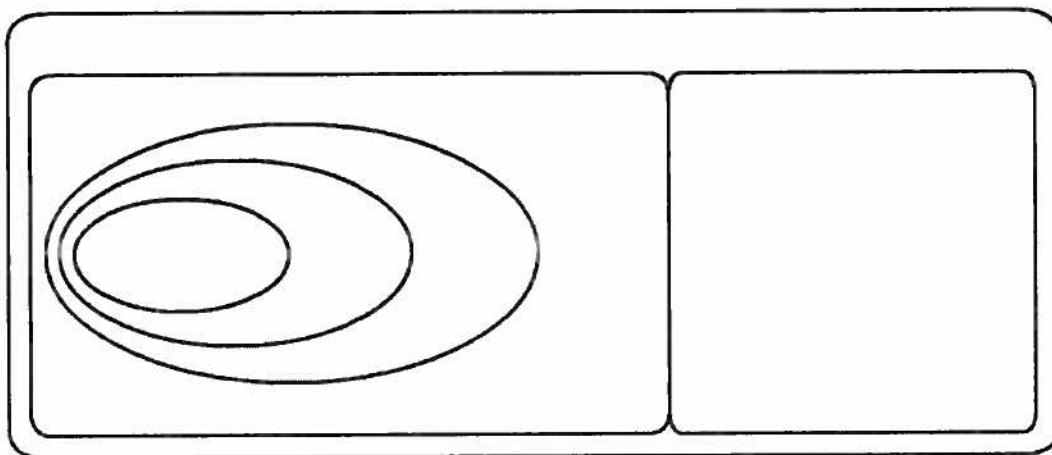
Numbers

1, 2, 3, 4, 5, 6, ...

Numbers

0, 1, 2, 3, 4, 5, 6, ...

... -3, -2, -1, 0, 1, 2, 3, ...



1.2 Real Numbers

Imaginary Numbers

Classify according to number type. (Some numbers may be of more than one type.)

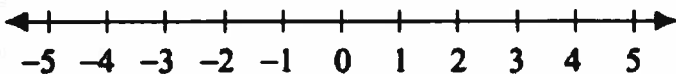
TRUE FALSE A rational is an integer.

TRUE FALSE A number is either a rational or an irrational, but not both.

1.3 Definition Absolute Value

$|4| =$

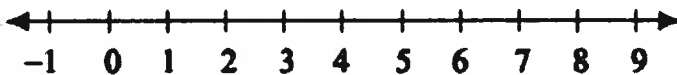
$|-4| =$



Example 1

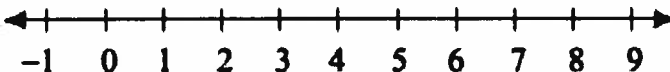
Determine each absolute value and then order them from least to greatest. Place the absolute values on the number line.

$$|-3|, |-8.7|, |1.75|, |0|, \left|6\frac{5}{8}\right|$$

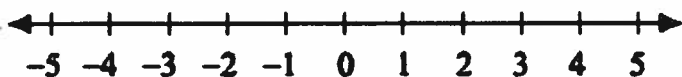
**1.4 Definition Absolute Value****Example 2**

Determine each absolute value and then order them from least to greatest. Place the absolute values on the number line.

$$|-4.2|, \left|-7\frac{1}{3}\right|, |2.3|, \left|-1\frac{3}{5}\right|$$



1.5 Distance on a Number Line



Example 3

Use the absolute value to determine the distance between each pair of numbers and show the solution on the number line.

a) -4.7 , -9.3

b) $-2\frac{2}{3}$, $7\frac{1}{4}$

1.6 Absolute Values and Square Roots

1.7 Absolute Values and Square Roots

Example 4 Evaluate: $\sqrt{(7 - 11)^2}$

Example 5 Using the idea $\sqrt{x^2} = |x|$, evaluate the following.

a) $\sqrt{\left(7\frac{1}{8}\right)^2}$

b) $\sqrt{\left(-5\frac{3}{7}\right)^2}$

c) $\sqrt{(8.7 - 14.2)^2}$

1.8 Evaluating Absolute Values

Example 6 Evaluate the following.

a) $|3| - |-14|$

b) $7 - 4|5 - 8|$

$$c) \left| -3(2-5)^2 + 9 \right|$$

1.9 Evaluating Absolute Values

Example 7 Evaluate the following.

$$a) \left| -18 \right| - \left| -21 \right|$$

$$b) \left| -8 \right| - 2 \left| 6 - 11 \right|$$

$$c) \left| 13(-2) - (-4)^2 \right|$$

Lesson 2

Simplifying Radicals

2.1 Working with Radicals



For $\sqrt{16}$ what is the value of the radicand and the index?

$$\sqrt[3]{64}$$

What number multiplied by itself _____ times equals _____.

What is the index of $\sqrt[3]{64}$

What is the radicand of $\sqrt[3]{64}$

2.2 Property of Radicals

$$\sqrt{4} \times \sqrt{9}$$

$$\sqrt{4 \times 9}$$

What can you conclude about.....and

$$\sqrt{4} \times \sqrt{9}$$

$$\sqrt{4 \times 9}$$

Property of Radicals

Is there a way we can express $\sqrt{30}$ as a product of two radicals?

2.3 Property of Radicals

Simplify: $\sqrt{8}$

Example 1 Simplify: $\sqrt{72}$

2.4 Property of Radicals

Simplify: $\sqrt{12}$

Michaela

$$\sqrt{12} =$$

Aleah

$$\sqrt{12} =$$

Who is correct?

Simplify: $\sqrt{252}$

2.5 Property of Radicals

Example 2 Simplify the following and leave in simplest radical form.

a) $\sqrt{245}$

b) $-4\sqrt{32}$

c) $\sqrt{70}$

2.6 Entire – Mixed Radicals

Simplify: $\sqrt{20}$

Write $2\sqrt{5}$ as an entire radical.

2.7 Mixed – Entire Radicals

Example 3 Write each of the following as an entire radical.

a) $6\sqrt{5}$

b) $3\sqrt{5}$

c) $-5\sqrt{2}$

2.8 Simplifying Radicals

Perfect Cubes

$$2^3 =$$

Simplify: $\sqrt[3]{16}$

Example 4 Simplify the following and leave in simplest radical form.

a) $\sqrt[3]{320}$

b) $\sqrt[3]{108}$

2.9 Entire Radicals

Write as an entire radical: $2\sqrt[3]{5}$

Example 5 Write the following as an entire radical.

a) $4\sqrt[3]{7}$

b) $10\sqrt[3]{2}$

2.10 Simplifying Radicals

$a^2 \times a^2 =$

$a^4 =$

$\sqrt{2^2} =$

$\sqrt{3^2} =$

$\sqrt{4^2} =$

$\sqrt{(756)^2} =$

$\sqrt{a^2} =$

$\sqrt{(x+1)^2} =$

Simplify: $\sqrt{a^4}$

Simplify: $\sqrt{a^7}$

2.11 Simplifying Radicals

Example 6 Simplify the following and write in simplest radical form

$$\sqrt{52a^6b^3}$$

2.12 Simplifying Radicals

Example 7 Write each radical as a mixed radical in simplest form.

a) $\sqrt{18a^3b^5}$

b) $\sqrt{24a^8b^7}$

2.13 Simplifying Radicals

Simplify: $\sqrt[3]{a^3}$

Simplify: $\sqrt[3]{a^6}$

Simplify: $\sqrt[3]{a^8}$

Example 8 Write each radical as a mixed radical in simplest form.

a) $\sqrt[3]{24a^5}$

b) $2\sqrt[3]{-54a^4b^7}$

c) $\sqrt[4]{48a^5b^7}$

2.14 Simplifying Radicals

Express as a mixed radical in simplest form.

$$\sqrt[3]{8a}$$

Write this mixed radical as an entire radical.

$$2\sqrt[3]{a}$$

Example 9 Write each mixed radical as an entire radical.

a) $5a\sqrt[3]{3a^2}$

b) $-3a^2b\sqrt[3]{2ab}$

c) $-5a^3b\sqrt[4]{2ab}$

2.15 Ordering Radicals

Order these numbers from least to greatest.

14, 9, 2, 7

Order these numbers from least to greatest.

$14x$, $9x$, $2x$, $7x$

Order these numbers from least to greatest.

$9\sqrt{3}$, $6\sqrt{3}$, $11\sqrt{3}$, $7\sqrt{3}$

Without using a calculator, order the following from least to greatest.

$5\sqrt{2}$, $4\sqrt{3}$, $3\sqrt{5}$, $2\sqrt{11}$

Lesson 3

Adding and Subtracting Radicals

3.1 Adding & Subtracting Radicals

$$\sqrt{3} + \sqrt{3} =$$

$$\square + \square =$$

$$\bigcirc + \bigcirc =$$

$$\sqrt{9} + \sqrt{9} =$$

$$\sqrt{7} + \sqrt{5} =$$

Adding Like Terms

$$2x + 3x =$$

$$2x + 3y =$$

Can Only Add Like Radicals

$$\sqrt{3} + \sqrt{3}$$

$$3\sqrt{7} + 5\sqrt{7}$$

3.2 Adding & Subtracting Radicals

$$\sqrt[3]{7} + 2\sqrt{7}$$

Example 1 Simplify and write in simplest radical form.

$$4\sqrt{11} - 6\sqrt{3} - 7\sqrt{11} + 5\sqrt{3}$$

Are $\sqrt{8}$ and $\sqrt{2}$ like radicals?

$$\sqrt{8} =$$

Example 2 Simplify and write in simplest radical form.

$$\sqrt{20} - \sqrt{45}$$

3.3 Adding & Subtracting Radicals

Example 3 Simplify and write in simplest radical form.

a) $2\sqrt{72} + \sqrt{20} - 5\sqrt{128}$

$$\text{b) } 2\sqrt{12} + 3\sqrt{112} - \sqrt{63} - 4\sqrt{75}$$

3.4 Adding & Subtracting Radicals

$$\sqrt[3]{7} + 2\sqrt{7}$$

$$\sqrt[3]{7} + 2\sqrt[3]{7}$$

Can you add these radicals?

Are $\sqrt[3]{16}$ and $\sqrt[3]{2}$ like radicals?

Example 4 Simplify and write in simplest radical form.

$$5\sqrt[3]{-16} + 4\sqrt[3]{250}$$

3.5 Adding & Subtracting Radicals**Example 5** Simplify and write in simplest radical form.

$$5\sqrt[3]{40} - 6\sqrt[3]{-135}$$

3.6 Adding & Subtracting with Variable Radicands**Example 6** Simplify the following.

$$7\sqrt{x} + 6\sqrt{x} - 11\sqrt{x}$$

Are these two expressions like radicals?

$$3a\sqrt{2a}, 7\sqrt{2a^3}$$

Example 7 Simplify and write in simplest radical form.

a) $7x\sqrt{x} - 5\sqrt{x^3}$

b) $2\sqrt{7a^2} - 4a\sqrt{28} + 5\sqrt{63a^2}$

c) $7b\sqrt{98a^3b} + 2a\sqrt{18ab^3} - 4b\sqrt{32a^3b}$

3.7 Adding & Subtracting with Variable Radicands

Can we add these radicals?

$$4b\sqrt[3]{2a} + 7b\sqrt[3]{5a}$$

Can we add these radicals?

$$-5a\sqrt[3]{2a} + 9b\sqrt[3]{2a}$$

Can we add these radicals?

$$3a\sqrt[3]{7b} - 8a\sqrt{7b}$$

Can we add these radicals?

$$3b\sqrt[3]{2a} - 2b\sqrt[3]{16a}$$

Example 8 Simplify the following.

a) $5b\sqrt[3]{32b} - 2\sqrt[3]{108b^4}$

b) $4\sqrt[3]{40m^4n} - 2m\sqrt[3]{135mn} + 7\sqrt[3]{320m^4n}$

3.8 More Than One Set of Like Radicals

$$5\sqrt{a} + 2\sqrt{b}$$

Example 9 Simplify the following.

a) $7\sqrt[3]{2a} - 9\sqrt{2a} + 4\sqrt[3]{2a} + 2\sqrt{2a}$

b) $3\sqrt{18b^3} - 9a\sqrt{48a^3} + 4\sqrt{12a^5} + 2b\sqrt{72b}$

Lesson 4

Multiplying Radicals

4.1 Multiplying Radicals

$$2\sqrt{3} \times 5\sqrt{7} =$$

Example 1 Simplify

$$2\sqrt{2} \times 3\sqrt{6}$$

Example 2 Simplify the following and leave in simplest radical form.

a) $5\sqrt{10} \times 4\sqrt{6}$

b) $3\sqrt{5} \times 2\sqrt{15}$

4.2 Multiplying Radicals

Example 3 Simplify $3\sqrt{7}(2\sqrt{14} - \sqrt{5})$

Example 4 Simplify the following and leave in simplest radical form.

$$2\sqrt{3}(2\sqrt{6} + 3\sqrt{8})$$

4.3 Multiplying Radicals

Example 5 Simplify and write in simplest radical form.

$$(\sqrt{2} + \sqrt{6})(\sqrt{3} - \sqrt{5})$$

Example 6 Simplify the following and leave in simplest radical form.

$$(2\sqrt{3} - 5)(3\sqrt{2} + \sqrt{10})$$

4.4 Multiplying Radicals

$$\sqrt{3} \times \sqrt{3} =$$

$$\sqrt{5} \times \sqrt{5} =$$

Without a calculator simplify:

$$\sqrt{1127} \times \sqrt{1127} =$$

$$2\sqrt{3} \times 2\sqrt{3}$$

Simplify: $5\sqrt{2} \times 5\sqrt{2}$

Example 7 Simplify the following and leave in simplest radical form.

$$(5\sqrt{3} - \sqrt{7})(5\sqrt{3} + 4\sqrt{7})$$

4.5 Multiplying Radicals

Example 8 Simplify the following and leave in simplest radical form.

a) $(2\sqrt{5} - 3\sqrt{7})(\sqrt{5} + 2\sqrt{7})$

$$b) (3\sqrt{a} - 2\sqrt{b})(5\sqrt{a} - 3\sqrt{b})$$

4.6 Multiplying Radicals

$$2^2 =$$

$$a^2 =$$

$$(a + b)^2 =$$

Common mistake!!

Example 9 Simplify the following and leave in simplest radical form.

$$(3\sqrt{5} + 2\sqrt{6})^2$$

4.7 Multiplying Radicals

Example 9 Simplify the following and leave in simplest radical form.

a) $(5\sqrt{2} - 3\sqrt{6})^2$

b) $(7\sqrt{x} + 4\sqrt{y})^2$

Lesson 5

Dividing and Rationalizing Radicals

5.1 Dividing Radicals

Property of Radicals

$$\sqrt{a} \times \sqrt{b} =$$

$$\frac{\sqrt{a}}{\sqrt{b}} =$$

Example 1 Simplify

a) $\frac{\sqrt{18}}{\sqrt{3}} =$

b) $\frac{6\sqrt{10}}{3\sqrt{2}} =$

c) $\frac{2\sqrt{6}}{5\sqrt{3}} =$

5.2 Dividing Radicals

Example 2 Simplify the following and write in simplest radical form.

a) $\frac{3\sqrt{20}}{2\sqrt{10}}$

b) $\frac{3\sqrt{12}}{6\sqrt{3}}$

c) $\frac{\sqrt{24}}{7\sqrt{12}}$

5.3 Rationalize the Denominator

$$\frac{\sqrt{21}}{\sqrt{3}}$$

$$\frac{\sqrt{11}}{\sqrt{2}}$$

Example 3 Simplify and rationalize the denominator.

$$\frac{3\sqrt{5}}{2\sqrt{12}}$$

Example 4 Simplify.

$$\frac{14\sqrt{3}}{2\sqrt{28}}$$

5.4 Dividing Radicals

Example 5 Simplify.

a)
$$\frac{3\sqrt{6}}{7\sqrt{20}}$$

$$\text{b) } \frac{3\sqrt{135a^5}}{\sqrt{21a^3}}$$

$$\text{c) } \frac{6\sqrt[3]{4n^7}}{\sqrt[3]{14n}}$$

5.5 Rationalize the Denominator

Example 6 Rationalize the denominator

$$\frac{\sqrt{6}-4}{\sqrt{5}}$$

Example 7 Rationalize the denominator

$$\frac{3\sqrt{2}+6}{\sqrt{2}}$$

5.6 Conjugates

$$(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})$$

$$(\sqrt{11} + \sqrt{3})(\sqrt{11} - \sqrt{3})$$

Example 8 Quickly simplify the following.

a) $(\sqrt{6} + \sqrt{13})(\sqrt{6} - \sqrt{13})$

b) $(\sqrt{14} - 3)(\sqrt{14} + 3)$

c) $(2\sqrt{3} - 3\sqrt{5})(2\sqrt{3} + 3\sqrt{5})$

5.7 Rationalizing the Denominator

$$\frac{5}{\sqrt{3}}$$

$$\frac{3}{\sqrt{7} + \sqrt{3}}$$

Example 9 Simplify the following and leave in simplest radical form.

Rationalize all denominators.

a)
$$\frac{\sqrt{5}}{3\sqrt{10} - 2\sqrt{5}}$$

b)
$$\frac{5\sqrt{3} - 3\sqrt{5}}{\sqrt{5} - \sqrt{3}}$$

Lesson 6
Solving Radical Equations

6.1 Restrictions on Radicands

Can we take the square root of negative?

$$\sqrt{-9} =$$

Can we take the square root of zero?

$$\sqrt{0}$$

What restriction must be on

$$\sqrt{\text{radicand}}$$

Example 1 For which values of the variable make the radical defined?

a) $\sqrt{3x^2}$

b) $\sqrt[3]{x}$

Example 2 For which values of the variable make the radical defined?

a) $\sqrt{-32a^3}$

b) $\sqrt[3]{-12y^5}$

c) $\sqrt[4]{8x}$

6.2 Restrictions on Radicands

For which values of the variable make the radical defined?

$$\sqrt{a-1}$$

Example 3 For which values of the variable make the radical defined?

a) $\sqrt{x+7}$

b) $\sqrt{3x+2}$

c) $\sqrt{5-x}$

6.3 Radical Equations

$$\sqrt{\quad}$$

$$\sqrt{x+2} + 4 = 13$$

Example 4 Solve: $\sqrt{x+9} = 7$

6.4 Extraneous Roots

Solve: $\sqrt{x-4} - 4 = 0$

Example 5 Solve: $\sqrt{x-4} + 4 = 0$

6.5 Solving Radical Equations

Example 6

Solve the radical equation algebraically

$$-3\sqrt{2x+1} + 5 = -4$$

Example 7

Solve the radical equation algebraically

$$\frac{2}{3}\sqrt{6x-3} + 5 = 11$$