

# UNIT 3: RATIONAL EXPRESSIONS

**Note-Taking Supplement**

**Student Package**

**Student's Name:** \_\_\_\_\_

- ☐ Once completed, submit this package to your Learning Facilitator.
- ☐ Click on the "Unit 3 Note-Taking Supplement" link in your Moodle course and follow the instructions to submit your request for marking.

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## Lesson 1

### Rational Expressions and Non-Permissible values

#### 1.1 Rational Numbers

Do you recall from previous Math courses what a Rational Number is?

**Example 1** Is  $\frac{1.27}{3}$  a rational number?

**Example 2** Is  $\frac{-3}{\sqrt{5}}$  a rational number?

#### 1.2 Rational Expression

**Rational Number**

**Rational Expression**

*There are three main types of polynomials that we will be working with.*

**Monomials**

**Binomials**

**Trinomials**

## 1.3 Review of Polynomials

Are the following expressions polynomials? If not, state why not.

a.  $3x^{-2} + 4x + 5$

b.  $\sqrt{2}x^3 - \sqrt{-5}x^2 + 1$

c.  $\sqrt{3}x^4 + \frac{5}{8}x^2 - 12$

## 1.4 Rational Expressions

Examples of Rational Expressions

Not a Rational Expression

## 1.5 Rational Expressions

**Example 3** Determine if the following are rational expressions or not. Give a reason for your answer.

a)  $\frac{x-7}{11}$

c)  $\frac{\sqrt[3]{a-3a}}{2a^4-9a^2+1}$

b)  $\frac{5^x+27}{x^2-9}$

d)  $\frac{2x^2-3x+4}{-3x^3+6x^2-x}$

## 1.6 Equivalent Rational Expressions

Find an equivalent rational number for each of the following by either multiplying or dividing the numerator and denominator by the SAME REAL NUMBER

$$\frac{2}{3}$$

$$\frac{42}{36}$$

Find an equivalent rational expression for each of the following by either multiplying or dividing the numerator and denominator by the same term

$$\frac{3a}{8b}$$

$$\frac{2}{5c}$$

Determine if the following pairs of rational numbers are equivalent, and if they are equivalent, list the number both the numerator and denominator was multiplied or divided by.

$$\frac{3}{4} \text{ and } \frac{15}{16}$$

Determine if the following pairs of rational expressions are equivalent, and if they are equivalent, list the term both the numerator and denominator was multiplied by.

$$\frac{2d}{7a} \text{ and } \frac{6d^3a}{21a^2d^2}$$

$$\frac{7}{3} \text{ and } \frac{49}{21}$$

$$\frac{(x-1)}{(x+4)} \text{ and } \frac{x^2-1}{x^2+5x+4}$$

$$\frac{14}{12} \text{ and } \frac{7}{6}$$

## 1.7 Non-Permissible Values

examples :  $\frac{x}{2}, \frac{4}{5x}, \frac{-3}{x-1}, \frac{2x}{x^2-3x-4}$

examples :  $\frac{1}{2}, \frac{4}{5}, -\frac{3}{2}$

Evaluate  $\frac{36}{2x}$  when  $x = 3$  | when  $x = 9$

What is the answer to this division:  $\frac{11}{0}$

Evaluate  $\frac{36}{2x}$  when  $x = 0$

## 1.8 Non-Permissible Values

Rational expressions are not defined for values of the variable that make the denominator 0.

d)  $\frac{x+1}{x^2-4x-5}$

These values are called:

Determine the non-permissible value(s) for the following rational expressions.

a)  $\frac{5}{x}$

b)  $\frac{3x}{x+1}$

c)  $\frac{4x+1}{3x+1}$

## 1.9 Non-Permissible Values

**Example 4** State the nonpermissible values for the variables in the denominators for each of the following.

a)  $\frac{7}{b}$

e)  $\frac{2x^2 - 3}{2x^2 - x - 3}$

b)  $\frac{2x}{4xy}$

c)  $\frac{x - 2}{4 - x}$

d)  $\frac{5y}{x^2 - 9}$

## 1.10 Equivalent Rational Expressions and Non-Permissible Values

$$\frac{x - 4}{2}$$

Evaluate when  $x = 5$

Evaluate when  $x = 7$

**1.11 Equivalent Rational Expressions and Non-Permissible Values****Example 2**

Use multiplication and division to write two equivalent forms of the rational expression  $\frac{2(x+3)}{(x+3)(x-4)}$

$$\frac{2(x+3)}{(x+3)(x-4)}$$

## Lesson 2

### ***Simplifying Rational Expressions***

## 2.1 Simplifying Rational Numbers, Expressions

$$\frac{9}{21} =$$

**Write in simplest form:**

12  
16

**Write in simplest form:**

$$\frac{15x^2y}{6xy^2}$$

## 2.2 Simplifying Rational Expressions

**Simplify the following monomial rational expressions.**

$$\frac{30xy}{6x}$$

$$\frac{-80x^3y^2}{16xy}$$

$$\frac{(-8x^3y^2)(6xy)}{(2xy^3)(-8x^2y)}$$

$$\frac{(-x^3y^4)^2(5xy)}{(2x^2y^3)(-3xy^2)^3}$$



## 2.3 Simplifying Rational Expressions

**Example 1** Simplify the following monomial rational expressions.

a) 
$$\frac{(3x^4y^5)(14x^2y)}{(7x^2y^3)(2x^2y^1)}$$

b) 
$$\frac{(-2x^2y^4)^4(5x^3y)^2}{(30x^2y^3)(-2x^3y^2)^3}$$

## 2.4 Simplifying Rational Expressions

Simplify rational expressions that involve binomials.

Take a common factor out of the numerator.

$$\frac{12a - 18b}{3}$$

$$\frac{8a^2b - 22ab^2}{-6ab}$$

## 2.5 Simplifying Rational Expressions

**Example 2** Simplify the following rational expression.

$$\frac{a^3}{a^2} =$$

a)  $\frac{14a^3b^2 - 42ab^4}{7a^2b}$

$$\frac{a^2}{a^3} =$$

$$\frac{x^4}{x^9}$$

## 2.6 Simplifying Rational Expressions

$$\frac{2x-6}{2x}$$

**2.7 Simplifying Rational Expressions****Example 3** Simplify the following.

a) 
$$\frac{3(x-7)(x+4)}{9(x+4)}$$

b) 
$$\frac{7a+14b}{3a+6b}$$

c) 
$$\frac{12x^2y^2-9y^4}{15xy^2+6y^3}$$

**2.8 Simplifying Rational Expressions****Example 4** Simplify the following.

$$\frac{x+2}{x^2-3x-10}$$

**Example 5** Simplify the following.

a) 
$$\frac{x^2-5x-14}{x^2-49}$$

b) 
$$\frac{2x^2-5x-3}{x^2+6x-27}$$

**2.9 Simplifying Rational Expressions****Evaluate when:  $a = 5$  and  $b = 4$** 

$$\frac{a - b}{b - a}$$

**Example 6 Simplify the following.**

$$\frac{20x - 15y}{21y - 28x}$$

**Evaluate when:  $a = 13$  and  $b = 7$** 

$$\frac{a - b}{b - a}$$

**Simplify:** 
$$\frac{2a - b}{b - 2a}$$

**2.10 Simplifying Rational Expressions****Example 7 Simplify the following.**

$$\frac{a^3 - ab^2}{ab - a^2}$$

## 2.11 Simplifying Rational Expressions

**Example 8** Find a value for  $k$  that makes the rational expressions true.

$$\frac{2x^2 - 13x + k}{3x^2 - 14x - 5} = \frac{2x - 3}{3x + 1}$$

### Lesson 3

#### Multiplying and Dividing Rational Expressions

#### 3.1 Multiplying and Dividing Rational Numbers

**Multiplying Rational Numbers**

$$\frac{2}{7} \times \frac{3}{5}$$

$$\frac{10}{21} \times \frac{9}{25}$$

**Dividing Rational Numbers**

$$\frac{4}{27} \div \frac{14}{15}$$

$$\frac{5}{8} \times \frac{4}{3} \div \frac{10}{3}$$

#### 3.2 Multiplying Rational Expressions

Multiply the following rational expression.

$$\frac{3xy}{3z} \times \frac{9x^2z^3}{3y}$$

Using Exponent Laws

$$\frac{3xy}{3z} \times \frac{9x^2z^3}{3y}$$

**Example 1**

Simplify the following rational expressions.

$$\frac{4x^3y^5}{6xy^4} \times \frac{9xyz}{2y^2}$$

### 3.3 Multiplying and Dividing Rational Expressions

$$\frac{6ab^2}{6a-9} \times \frac{2a-3}{-18a^2b}$$

**Example 2** Simplify the following rational expressions.

$$\frac{4x^3y^2}{5(x+y)} \div \frac{-16x^2y^4}{3(x+y)}$$

### 3.4 Multiplying and Dividing Rational Expressions

**Example 3** Simplify each of the following for all permissible values of the variables.

a)  $\frac{6(x-5y)}{xy^2} \times \frac{y(x+3y)}{9(x-5y)}$

b)  $\frac{8ab}{4b^2 - a^2} \div \frac{4b^2}{3(a-2b)^2}$

### 3.5 Multiplying and Dividing Rational Expressions

**Example 4** Simplify the following.

$$\frac{x^2 - 2x - 15}{x^2 + 5x + 6} \times \frac{x^2 - 4x - 21}{x^2 - 4x - 5}$$

**Example 5** Simplify the following.

$$\frac{2x^2 - 9x - 5}{6x^2 - 7x + 2} \div \frac{4x^2 + 4x + 1}{4x^2 - 4x + 1}$$

### 3.6 Complex Fractions

Simplify the following complex rational expression.

$$\frac{2x + 2}{x^2 - 9x + 20}$$

$$\frac{4x}{x^2 + 2x - 35}$$

Write the same expression using a division sign.

Identify the parts of the complex rational expression where the restrictions on the variable  $x$  will be found.

Factor all the terms where possible, state restrictions and then simplify the complex rational expression.



**3.7 Complex Fractions****Example 6** Simplify the following.

$$\frac{\frac{3x^2 - x - 4}{3x^2 - 7x + 4}}{\frac{3 - 3x^2}{2x - 2}}$$

## Lesson 4

### Adding and Subtracting Rational Expressions

#### 4.1 Adding Rational Expressions

**To add or subtract rational numbers (fractions), you must have a COMMON DENOMINATOR**

**Perform the indicated operation for the following rational expressions.**

Perform the indicated operations for the following rational numbers.

$$\frac{2}{3} - \frac{7}{3} + \frac{1}{3}$$

$$\frac{2a}{4b} + \frac{3a}{4b} - \frac{5}{4b}$$

#### 4.2 Common Denominators

**To add or subtract rational numbers (fractions), you must have a COMMON DENOMINATOR**

**Perform the indicated operation for the following rational expressions.**

$$\frac{2}{3} + \frac{3}{4}$$

$$\frac{2}{3a} + \frac{3}{b}$$

**4.3 Common Denominators**

Perform the indicated operation for the following rational expressions.

$$\frac{2x + y}{3} - \frac{x - 2y}{2}$$

**Example 1** Simplify.

$$\frac{a}{2} - \frac{a + 4}{a + 5} + 3$$

**4.4 Adding Rational Expressions**

Perform the indicated operation for the following rational expressions.

$$\frac{3}{a - 2} + \frac{4}{a - 5}$$

**Example 2** Simplify.

$$\frac{4}{3x + 1} - \frac{2}{4x - 5}$$

**4.5 LCM & Common Denominators**

$$\frac{2}{a} + \frac{4}{a}$$

Would you multiply the denominators to find the CD?

$$\frac{2}{3a} + \frac{4}{a}$$

**4.6 Lowest Common Multiple**

Find the lowest common multiple for each of the following pairs of terms.

$$9xy^3 \text{ and } 6x^2y^2$$

$$(x+1)(x-3) \text{ and } 4(x-3)$$

$$(x+5)(x+3) \text{ and } (x+5)(x+5)$$

$$6x^2 + 11x - 10 \text{ and } 16x^2 - 100$$

**4.7 Adding and Subtracting Rational Expressions**

Perform the indicated operation for the following rational expressions.

$$\frac{3}{2x+6} + \frac{3}{2} - 4$$

**Example 3** Simplify.

$$\frac{4}{3x-12} - \frac{5}{2(x-4)^2} + 1$$

**4.8 Adding and Subtracting Rational Expressions**

Perform the indicated operation for the following rational expressions.

$$\frac{7b}{b^2 - b - 12} - \frac{4b}{b^2 + 2b - 3}$$

**Example 4** Simplify.

$$\frac{a-6}{a^2-11a+28} - \frac{a-5}{a^2-8a+7}$$

**Lesson 5**  
**Solving Rational Equations**

**5.1 Rational Expressions**

$$6 \div 0 =$$

$$\frac{6}{0}$$

$$\frac{6}{x}$$

Evaluate:  $\frac{6}{x}$ , when  $x = 2$

Can we evaluate  $\frac{6}{x}$ , when  $x = 7, .24, \pi, \frac{1}{7}$ ?

$x$  cannot equal what value?

Restrictions for rational expressions.

State the restrictions.

$$\frac{-7}{x+9}$$

$$\frac{3x}{7x+3}$$

**5.2 Rational Expressions****Example 1**

State all restrictions for the following rational expressions.

a)  $\frac{3x + 1}{11 - 5x}$

b)  $\frac{x - 3}{(x - 5)(2x + 1)}$

c)  $\frac{x + 1}{6x^2 - 13x - 5}$

**5.3 Rational Expressions**

Solve:  $\frac{1}{2}x + \frac{2}{3} = \frac{3}{5}$

Determine the Lowest Common Denominator of Rational Expressions

$$\frac{1}{2} + \frac{1}{3}$$

$$\frac{1}{x} + \frac{1}{x}$$

$$\frac{1}{5x} + \frac{1}{4x}$$

$$\frac{1}{a} + \frac{1}{ab}$$

$$\frac{1}{a^2} + \frac{1}{a}$$

$$\frac{4}{x} + \frac{5}{x-3}$$

$$\frac{3x}{x+2} + \frac{5x}{(x+2)(x-3)}$$

## 5.4 Solving Rational Equations

### Example 2

Solve the following rational equation and state any restrictions.

$$\frac{1}{2x} - \frac{1}{4x} = \frac{1}{6}$$



**5.5 Solving Rational Equalities****Example 3**

Solve the following rational equations and state any restrictions.

a)  $\frac{4}{3x} - \frac{2}{5} = \frac{1}{2x}$

b)  $\frac{4}{7x} - \frac{1}{2} = 1 + \frac{2}{x}$

**5.6 Solving Rational Equalities****Example 4**

Solve the following rational equations and state any restrictions.

$$\frac{x}{x-2} + \frac{2}{3} = \frac{2}{x-2}$$

**5.7 Solving Rational Equalities****Example 5**

Solve the following rational equations and state any restrictions.

$$\frac{4}{2x-1} - \frac{3}{5} = \frac{7x}{2x-1}$$

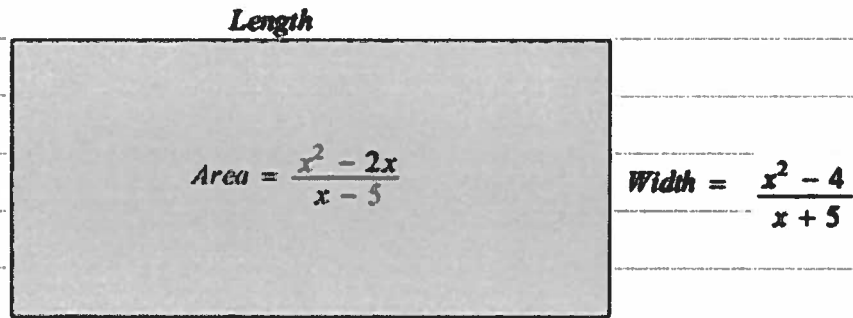
## Lesson 6

### Solving Problems involving Rational Expressions

#### 6.1 Rational Expressions

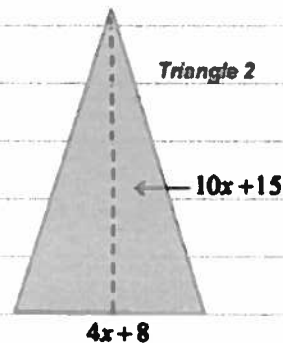
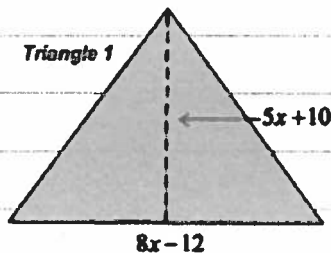
Given the area and the width of the rectangle shown below, find an expression for the length.

How would you find an expression for the length of the rectangle with the given information?



#### 6.2 Rational Expressions

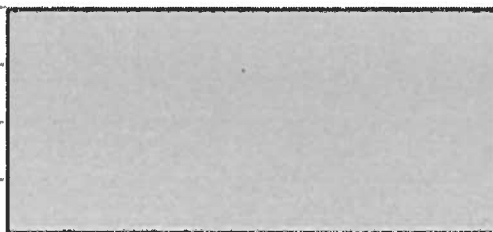
Write and simplify an expression that represents the ratio of the area of Triangle 1 to the area of Triangle 2.



**6.3 Rational Expressions**

Given the length and the width of the rectangle shown below, find an expression for the perimeter.

How would you find an expression for the perimeter of the rectangle?



$$\text{Width} = \frac{2}{6x+15}$$

$$\text{Length} = \frac{8}{4x^2 - 25}$$

**6.4 Rational Expressions****Example 1**

Jon walked from home to the local fitness center at a speed of 6 km/hr. On the trip back home he jogged at 15 km/hr. If both trips combined took 2 hours and 48 minutes, what is the distance from Jon's home to the fitness center?

**6.5 Rational Expressions****Example 2**

It takes an express train one hour longer to travel 240 kilometers than it does for a small plane to travel the same distance. If the plane travels twice as fast as the train, how fast does the plane travel?

**6.6 Rational Expressions****Example 3**

A car travels 420 kilometers in the same time as it takes a truck to travel 300 kilometers. If the car is travelling 30 km/hr faster than the truck, what is the car's speed?

**6.7 Rational Expressions****Example 4**

Jim travelled 150 km by bus and then 1350 km by plane. The speed of the plane was six times as fast as the bus. If the entire trip took 5 hours, what was the speed of the bus and the plane?

**6.8 Rational Expressions****Example 5**

It took a small plane 45 minutes longer to travel 300 km than it took a jet to travel the same distance. If the speed of the jet was three times as fast as the small plane, what was the speed of the small plane and the jet to the nearest kilometer per hour?