UNIT 4: QUADRATIC FUNCTIONS

Note-Taking Supplement

Student Package

Student's Name: _____

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Date submitted:

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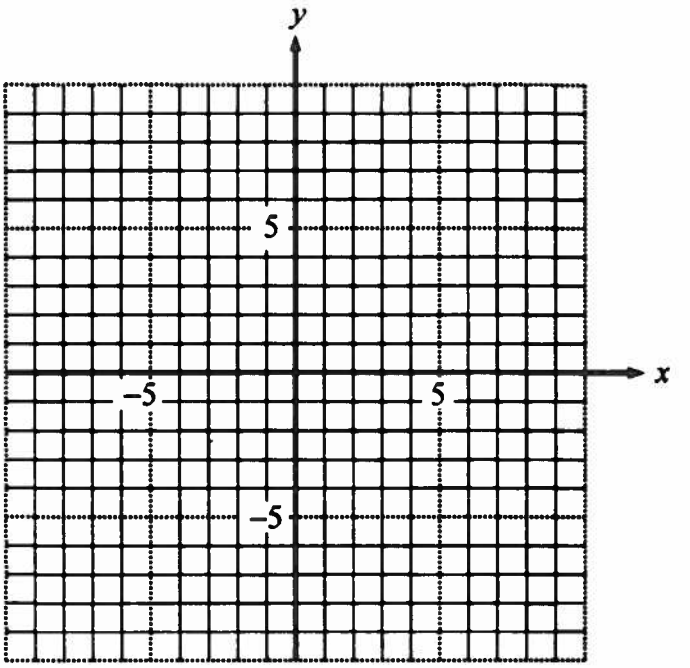
Lesson 1
Transformations - Translations

1.1 Graphing Quadratics

Quadratic Functions

Graph $y = x^2$

x	y



Domain: _____

Range: _____

Equation of the axis of symmetry:

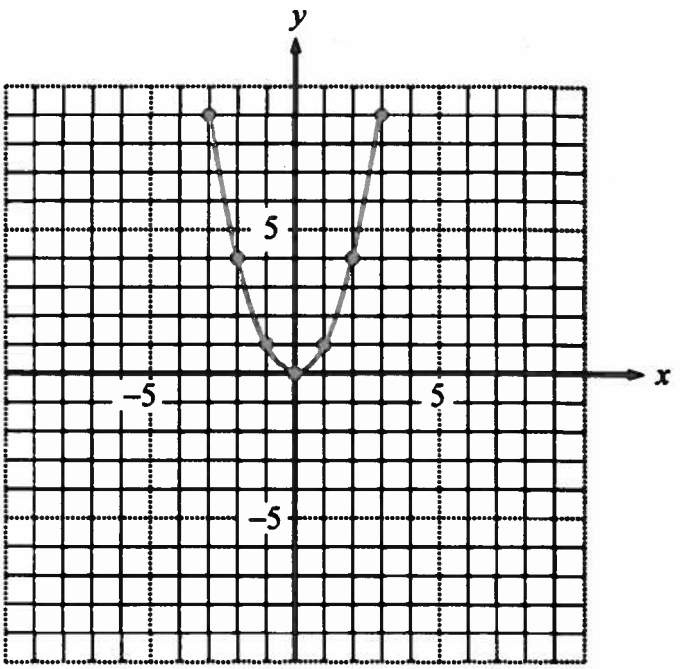
Coordinates of vertex:

1.2 Graphing Quadratics

$y = x^2$ *Model Parabola*

Graph $y = x^2 - 4$

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = x^2 - 4$.

_____

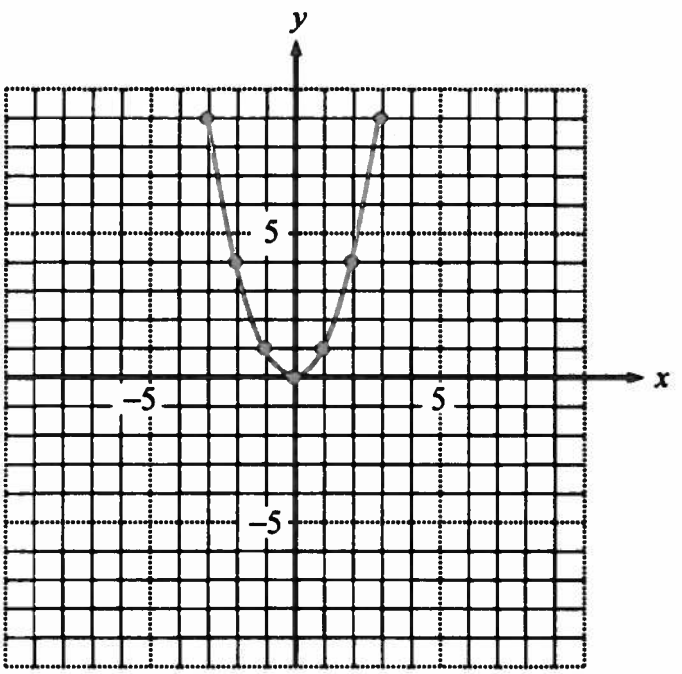
Domain: _____ Equation axis of symmetry:

Range: _____ Coordinates of the Vertex:

1.3 Graphing Quadratics

Graph $y = x^2 + 3$

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = x^2 + 3$.

_____

Domain: _____ Equation axis of symmetry:

Range: _____ Coordinates of the Vertex:

1.4 Graphing Quadratics

$y = x^2$ *Model Parabola*

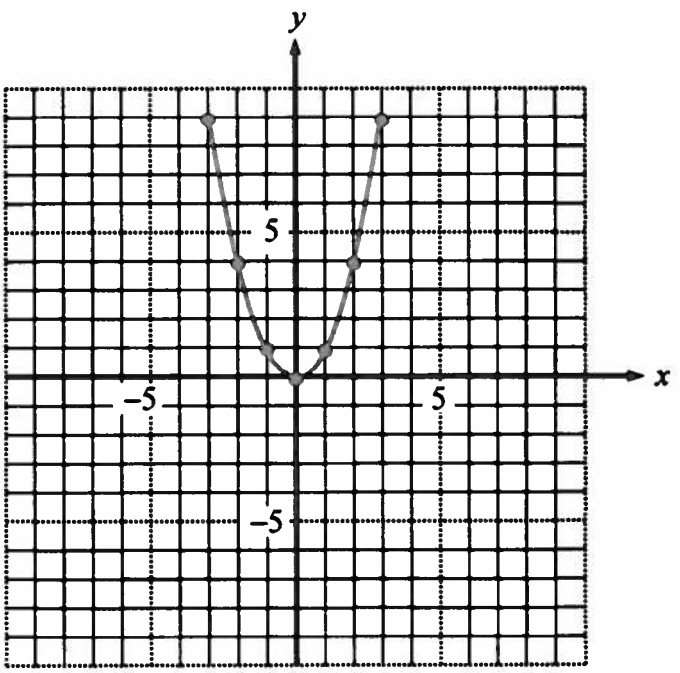
$y = x^2 - 4$ $\qquad$ $y = x^2 + 3$

$y = x^2 + k$

1.5 Graphing Quadratics

Graph $y = -x^2$

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = -x^2$.

Domain: _____ Equation axis of symmetry:

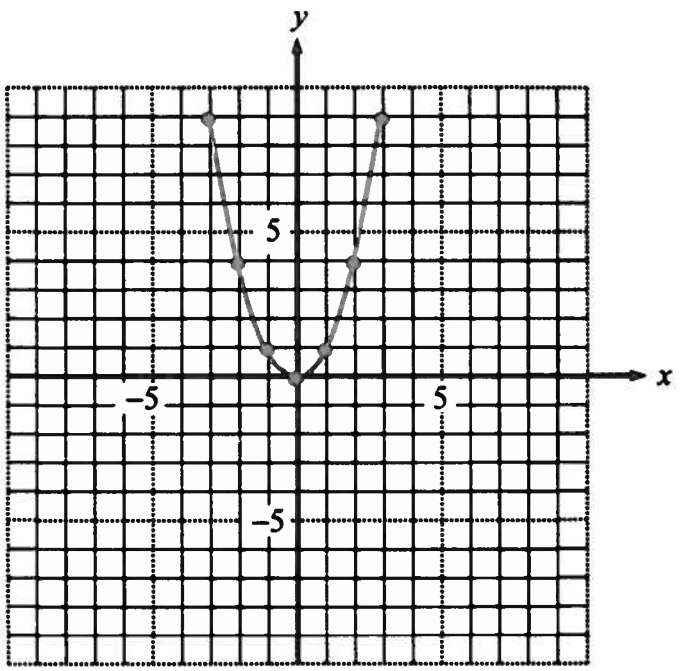
Range: _____ Coordinates of the Vertex:

$y = x^2$	$y = -x^2$

1.6 Graphing Quadratics

Graph $y = (x+4)^2$

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = (x+4)^2$.

_____

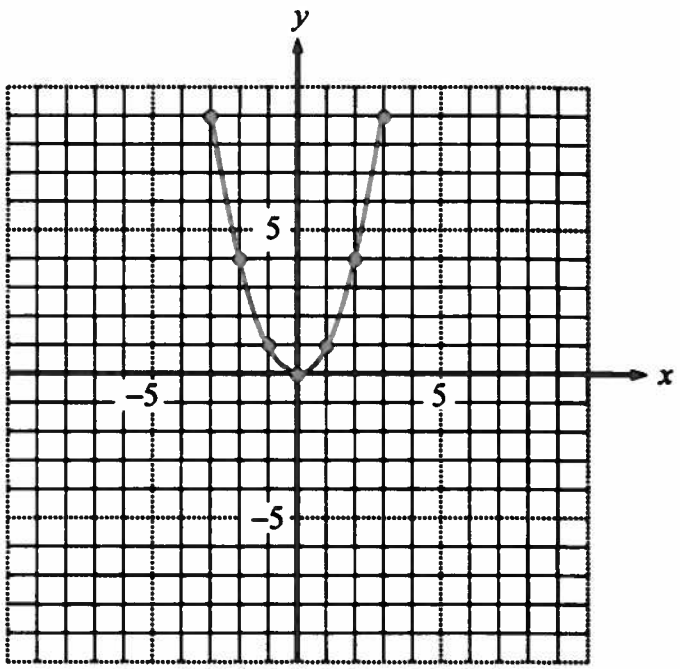
Domain: _____ Equation axis of symmetry:

Range: _____ Coordinates of the Vertex:

1.7 Graphing Quadratics

Graph $y = (x-7)^2$

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = (x-7)^2$.

_____

Domain: _____ Equation axis of symmetry:

Range: _____ Coordinates of the Vertex:

1.8 Graphing Quadratics

$y = x^2$ *Model Parabola*

$y = (x+4)^2$ $\qquad$ $y = (x-7)^2$

$y = (x+4)^2$ $\qquad$ $y = (x-7)^2$

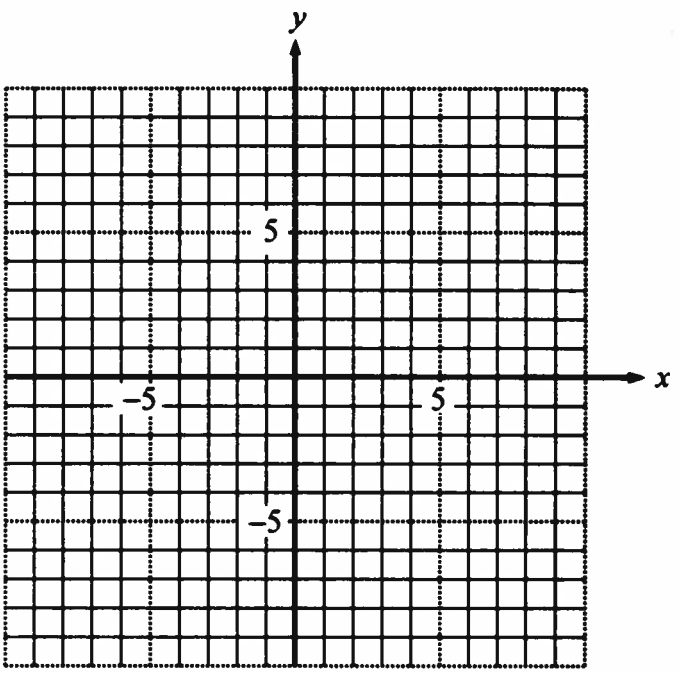
$y = (x-h)^2$ $\qquad$ $y = (x-h)^2$

What is the value of h to get $+4$? $\qquad$ What is the value of h to get -7?

$y = (x - h)^2$

1.9 Graphing Quadratics

Graph $y = (x + 5)^2 - 7$



$y = (x + 5)^2 - 7$

$y = (x - h)^2 + k$

Vertex: (,)

1.10 Graphing Quadratics

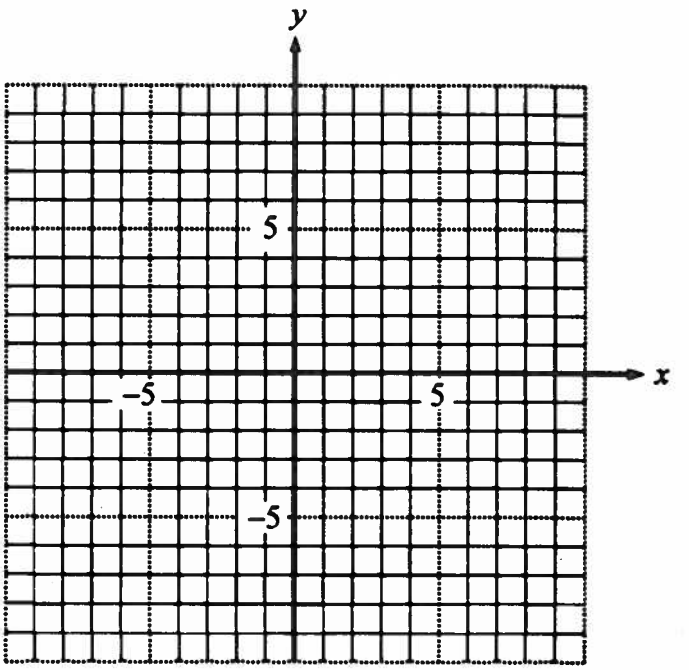
Graph $y = (x-4)^2 - 3$

Vertex: (,)

Equation axis of symmetry:

Direction of opening:

Domain: ***Range:***



1.11 Graphing Quadratics

The Maximum or Minimum Value of a Parabola

Graph $y = -(x+6)^2 + 4$

Vertex: (,)

When we talk about MAXIMUM or MINIMUM value

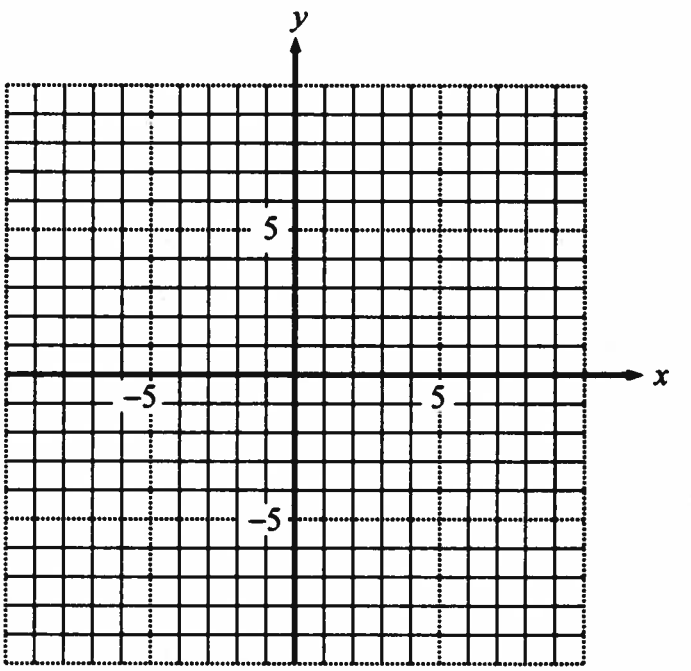
we mean MAXIMUM _____

or MINIMUM _____

This parabola has a _____ value.

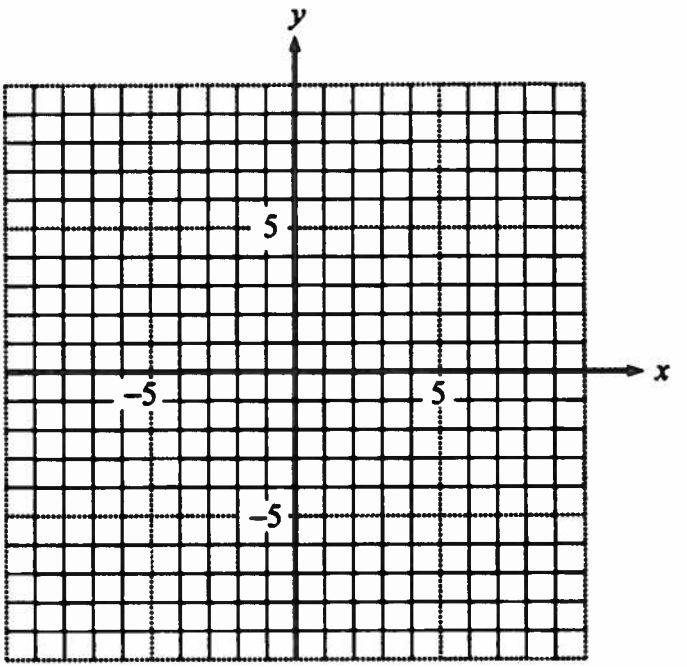
Minimum y-value of _____

This occurs when _____



Graph $y = (x-4)^2 - 7$

Vertex: (,)



This parabola has a _____ value.

Maximum y-value of _____

This occurs when _____

$y = -(x+6)^2 + 4$

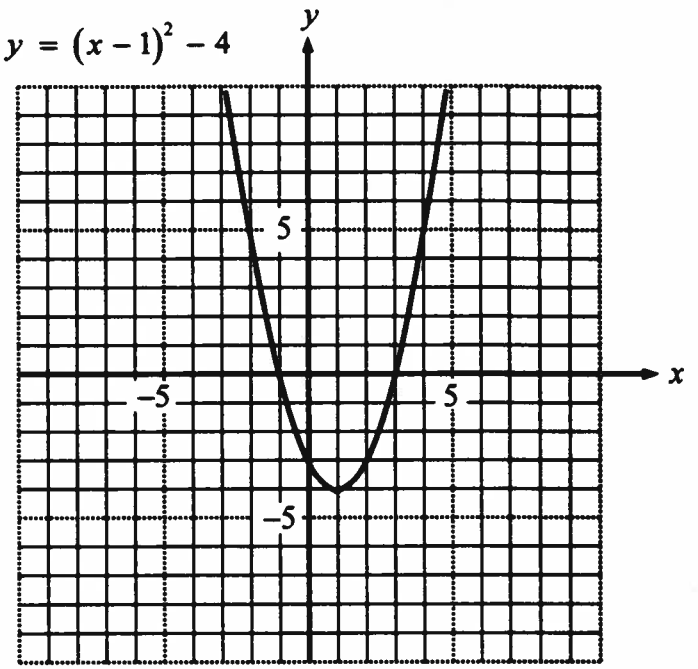
$y = (x-4)^2 - 7$

1.12 Graphing Quadratics

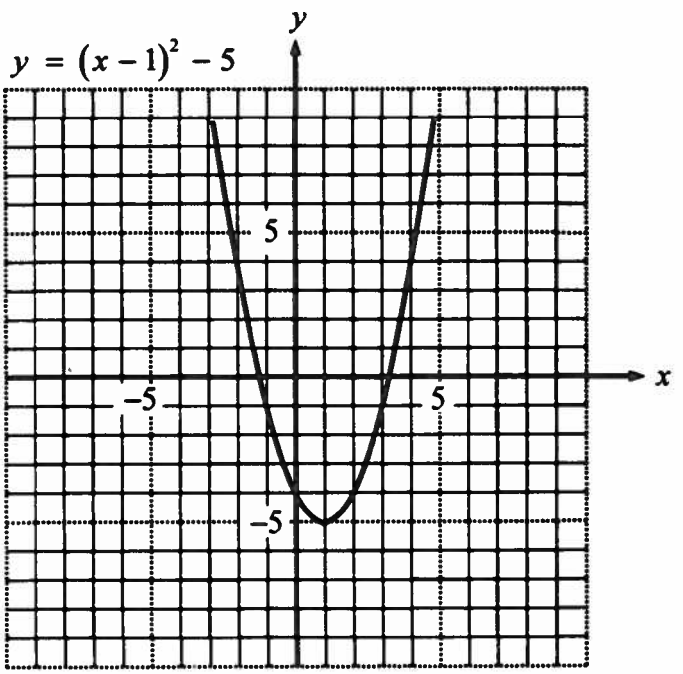
X-Intercepts

X-intercepts are where the graph crosses the x-axis

The value of the y-coordinate for any x-intercept is always _____.



Determine the X-intercepts of $y = (x-1)^2 - 5$

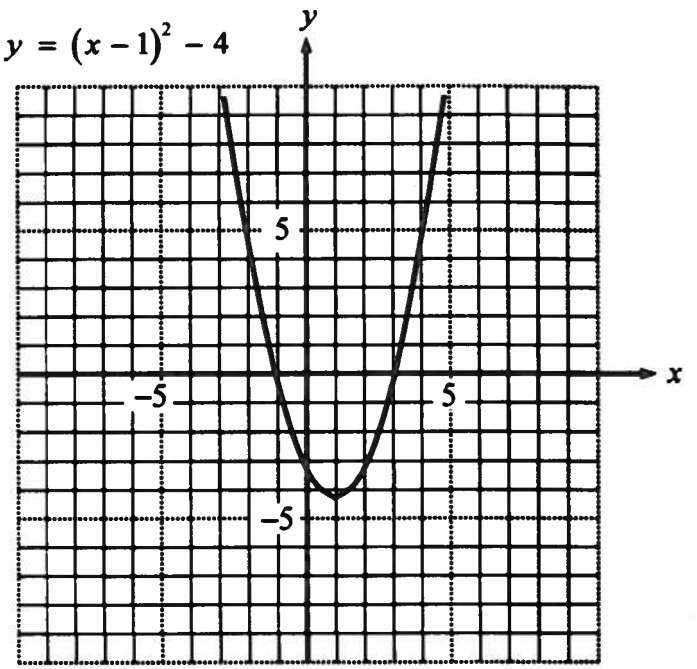


1.13 Graphing Quadratics

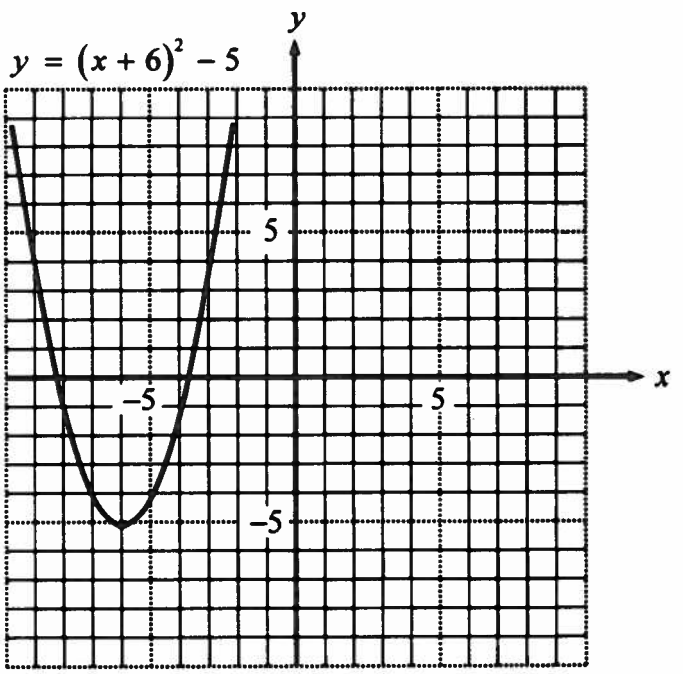
Y-Intercept

Y-intercept is where the graph crosses the y-axis

The value of the x-coordinate for any y-intercept is always _____.



Determine the Y-intercept of $y = (x+6)^2 - 5$



1.14 Graphing Quadratics

Example 1 The parabola $y = x^2$ is transformed as described below. Its image has the form $y = a(x-h)^2 + k$. Determine the value of a, h, and k for each transformation then write the equation in the form $y = a(x-h)^2 + k$.

a) Translate the parabola 11 units left , 4 units down and it opens up.

b) Translate the parabola 9 units right, 13 units up and it opens down.

Example 2 Sketch the graph for each parabola. Label the vertex and the coordinates of two other points and then state:

a) equation of axis of symmetry
b) direction of opening
c) maximum or minimum value
d) coordinates of the y-intercept and x-intercepts (exact values)
e) domain and range

i) $y = (x-4)^2 - 2$

ii) $y = -(x+5)^2 + 7$

Lesson 2
Compressions - Expansions

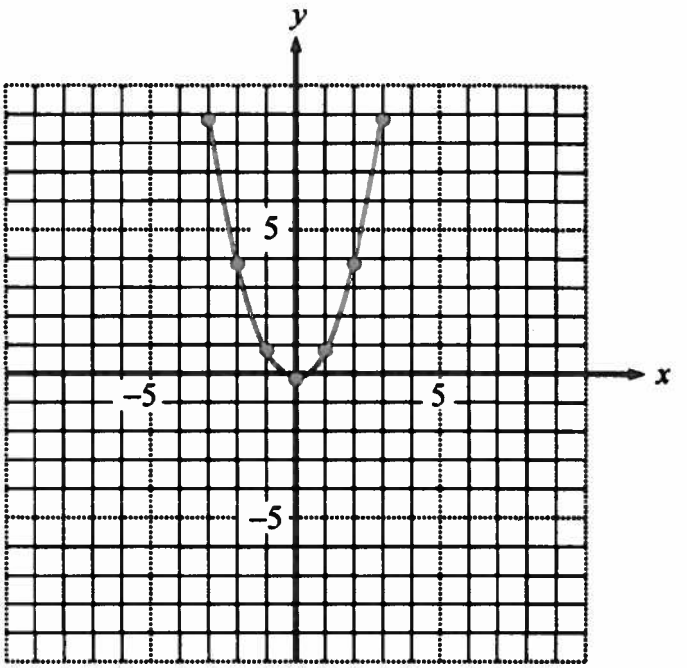
2.1 Expansions and Compressions

Model Parabola

$y = x^2$ $y = 2x^2$

x	y
3	9
2	4
1	1
0	0
−1	1
−2	4
−3	9

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = 2x^2$.

_____

Compare the y-values of $y = x^2$ to the y-values of $y = 2x^2$.

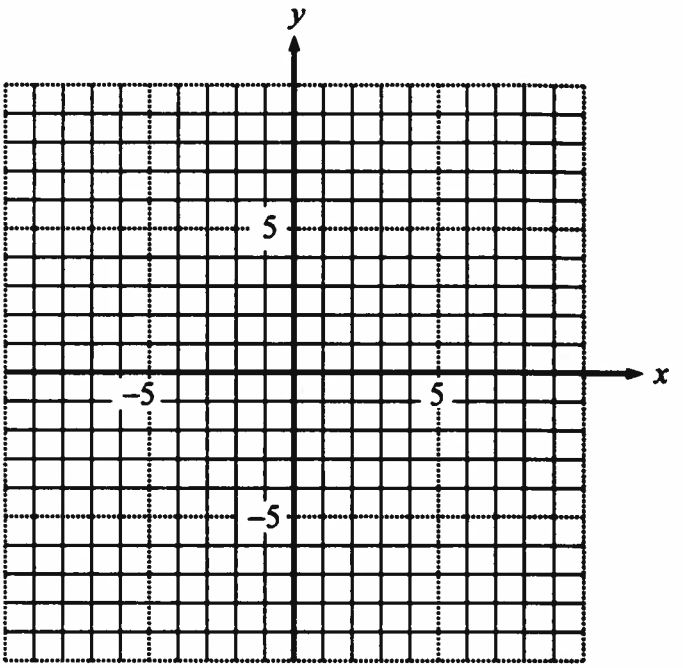
What do you notice? _____

$y = 2x^2$

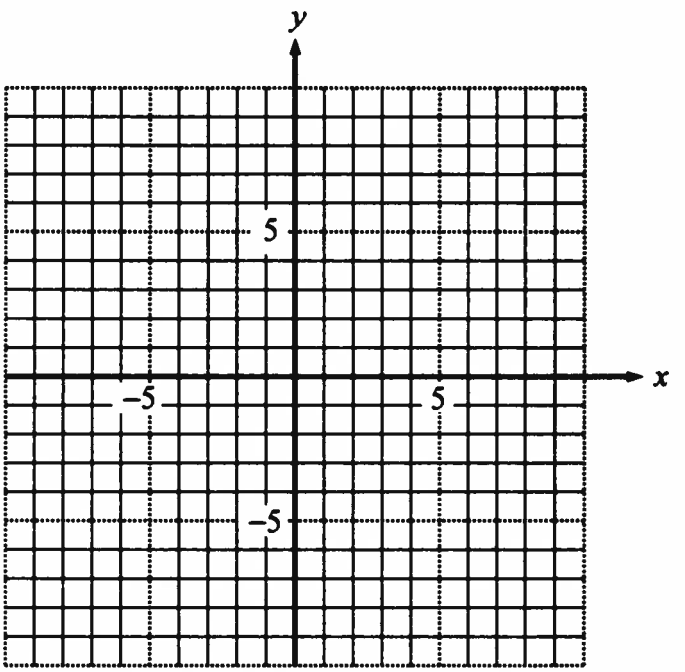
$y = ax^2$

2.2 Expansions and Compressions

Graph $y = 4x^2$ without making a table of values.



Graph $y = 3x^2$ without making a table of values.



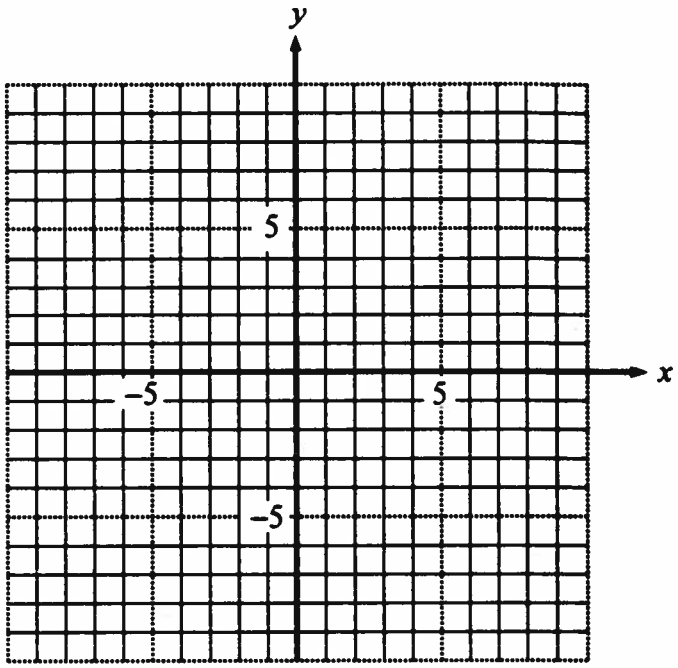
2.3 Expansions and Compressions

Model Parabola

$y = x^2$ $\quad$ $y = \frac{1}{2}x^2$

x	y
3	9
2	4
1	1
0	0
−1	1
−2	4
−3	9

x	y



Describe what has happened to the graph of $y = x^2$ when we graph $y = \frac{1}{2}x^2$.

_____

Compare the y-values of $y = x^2$ to the y-values of $y = 2x^2$.

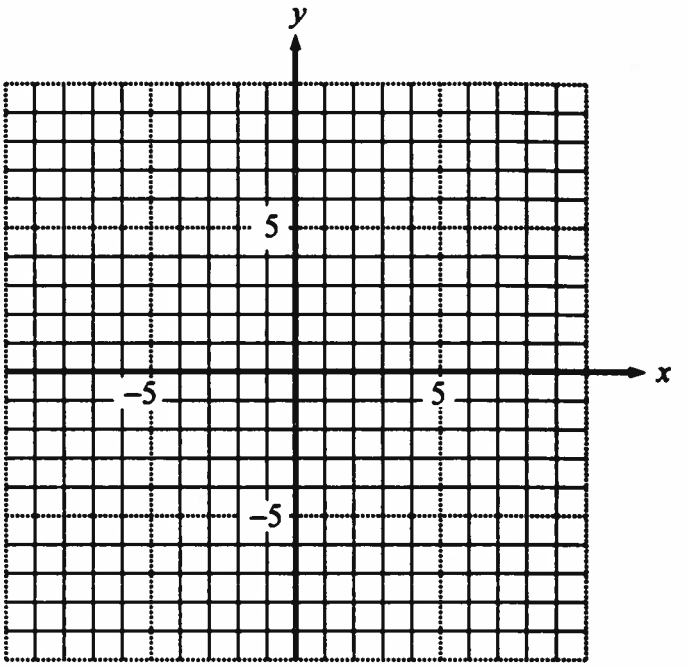
What do you notice? _____

$y = \frac{1}{2}x^2$

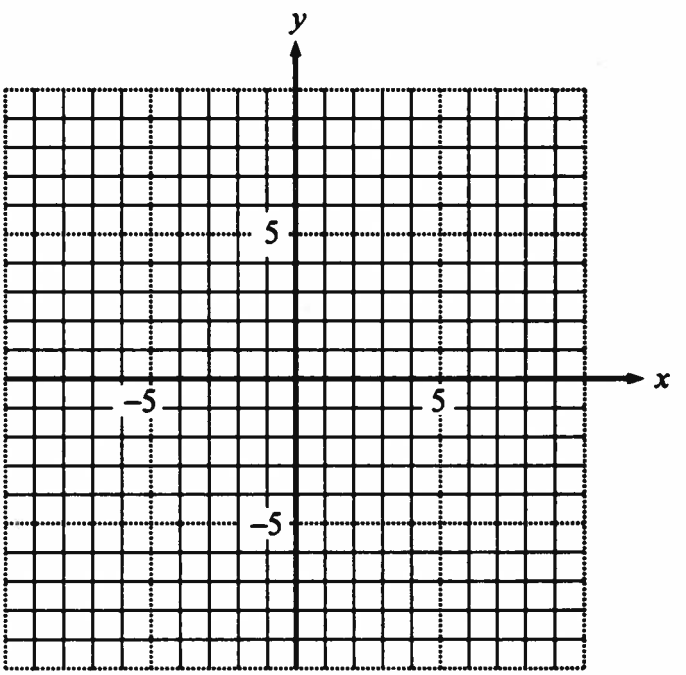
$y = ax^2$

2.4 Expansions and Compressions

Graph $y = \frac{1}{4}x^2$ without making a table of values.



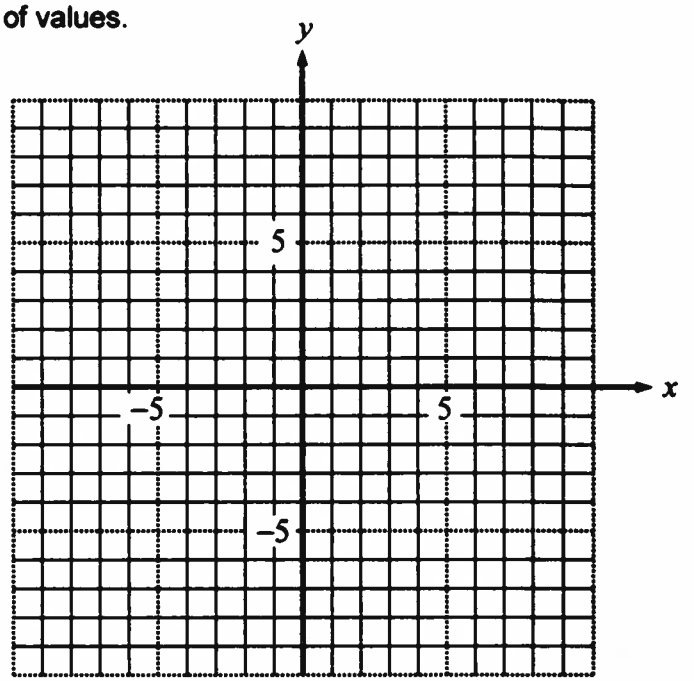
Graph $y = \frac{1}{3}x^2$ without making a table of values.



2.5 Expansions and Compressions

Example 1

Graph $y = -2(x-3)^2 + 1$ without making a table of values.



Equation of axis of symmetry:

_____value of _____ which occurs when _____.

Y-intercept:

X-Intercepts: exact values

Domain: _____

Range: _____

2.6 Expansions and Compressions

Example 2

Graph $y = -\frac{1}{5}(x+2)^2 + 6$ without making a table of values.

Then determine the following:

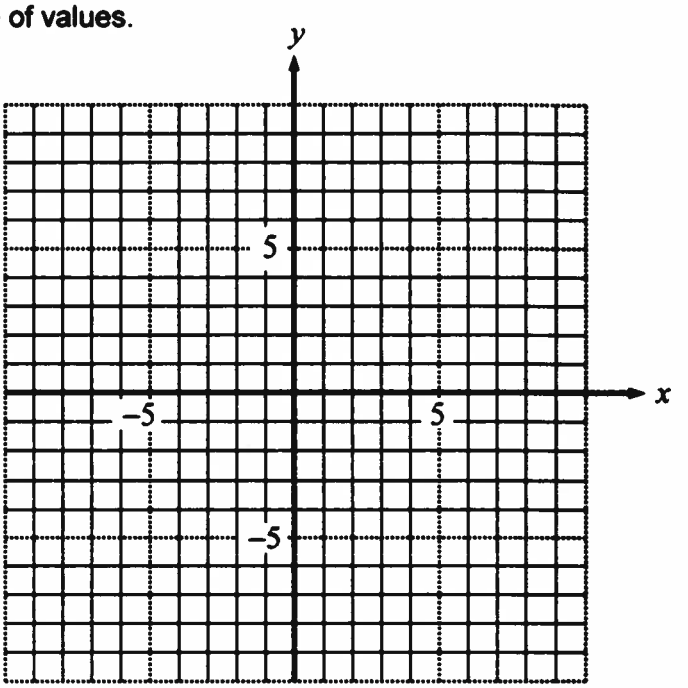
a) Equation of axis of symmetry.

b) Maximum or Minimum value.

c) Y-intercept

d) X-intercepts (exact values)

e) Domain and Range



2.7 Graphing Quadratics with Calculator

Example 3

The path a tennis ball takes from a players forehand ground stroke can be modeled by the function: $h(d) = -0.013(d-8)^2 + 1.3$, where $h(d)$ is the height of the ball and d is the horizontal distance the ball has traveled since it was struck.

a) What is the maximum height the tennis ball reaches?

b) How far has the ball traveled horizontally from where it was struck when it reaches its maximum height?

c) What was the height of the ball when it was struck?

d) How far did the ball travel horizontally from where it was struck to whre it landed inside the court?

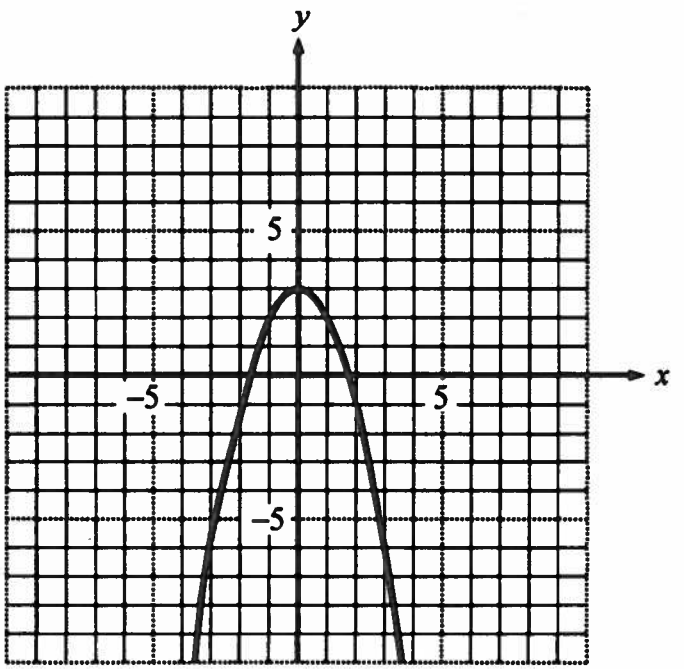
e) If the opponent is standing 12 meters away, at what height would she make contact with the ball to volley it back?

f) State what represents the domain and the range in this example, then list the domain and range.

Lesson 3
Determining Equations of Quadratic Functions

3.1 Equation from Graph

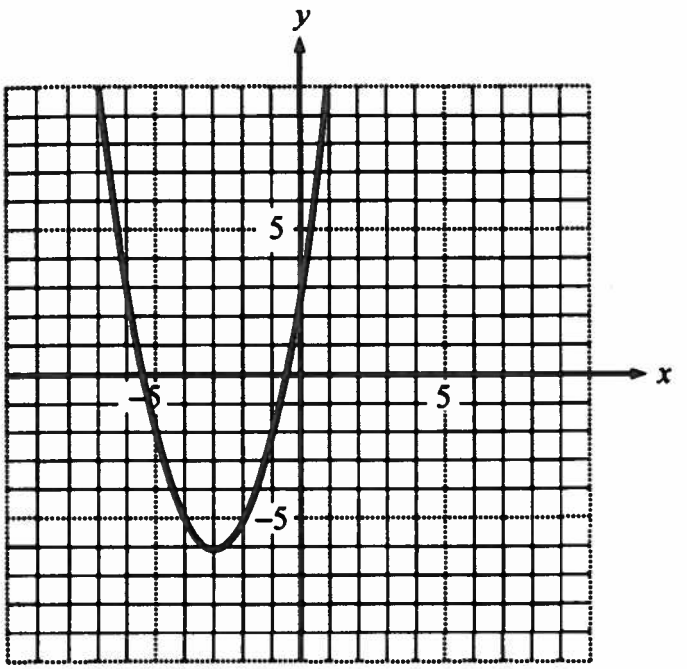
Given the graph of a quadratic function, determine its equation in the form $y = a(x - h)^2 + k$.



3.2 Equation from Graph

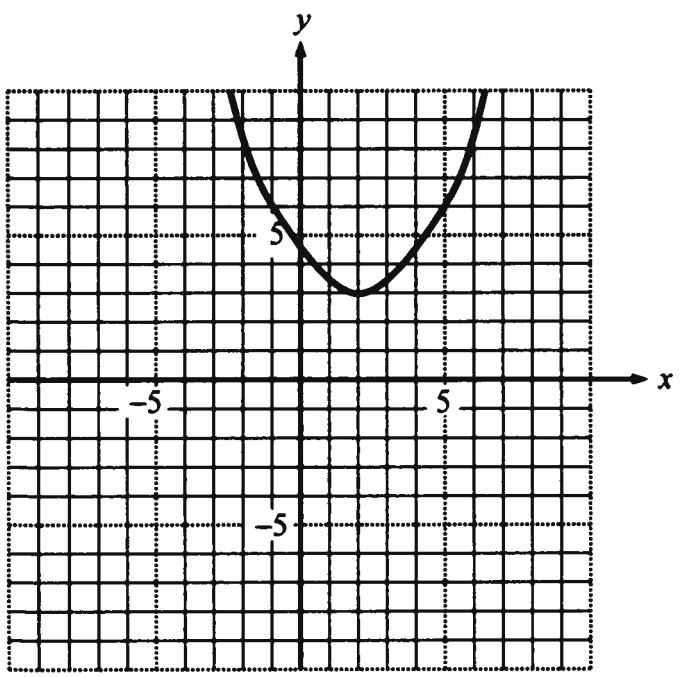
Example 1

Given the graph of a quadratic function, determine its equation in the form $y = a(x - h)^2 + k$.



3.3 Equation from Graph

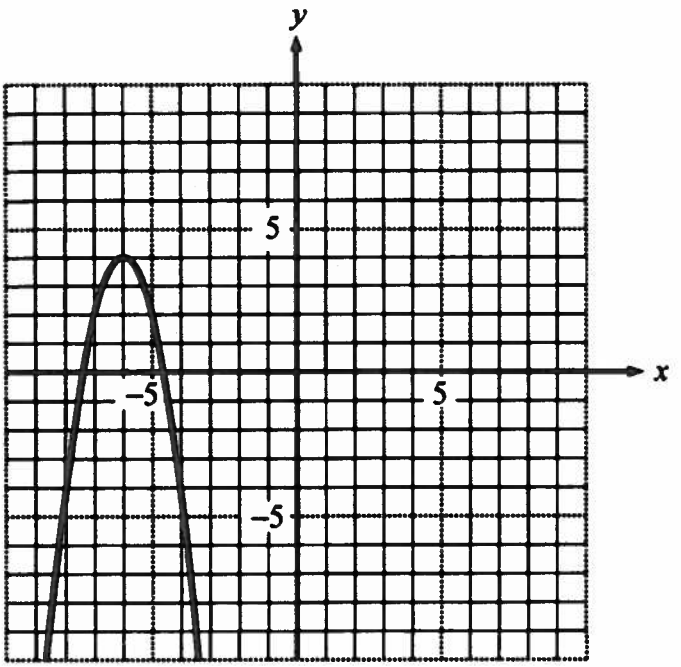
Given the graph of a quadratic function, determine its equation in the form $y = a(x - h)^2 + k$.



3.4 Equation from Graph

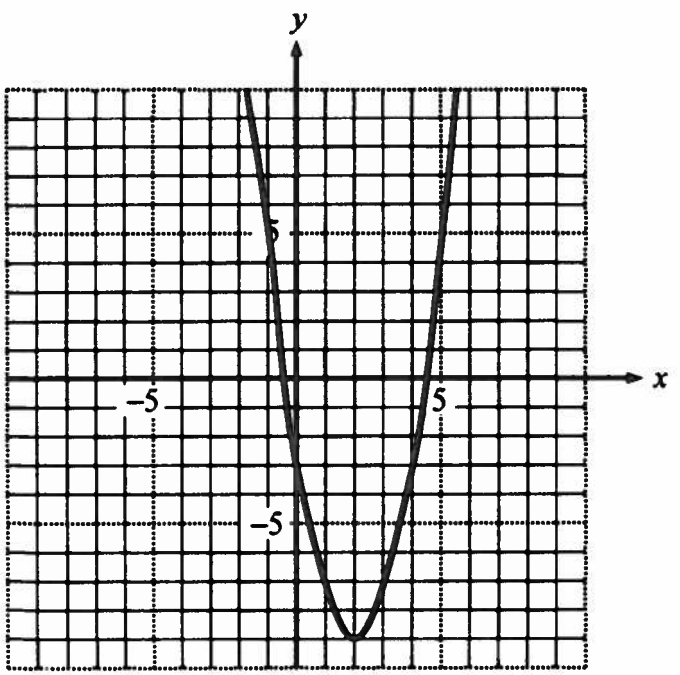
Example 2

Given the graph of a quadratic function, determine its equation in the form $y = a(x-h)^2 + k$.



3.5 Equation from Graph

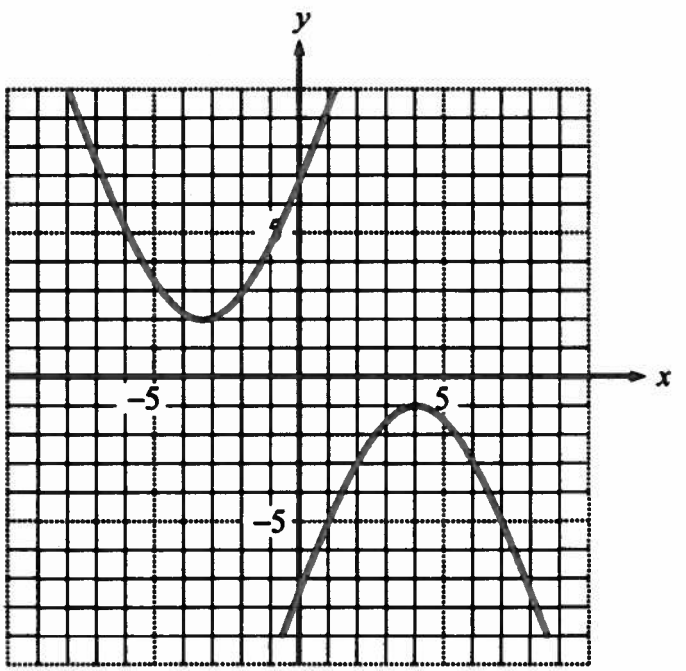
Given the graph of a quadratic function, determine its equation in the form $y = a(x-h)^2 + k$.



3.6 Equation from Details

Determine the equation of the parabola with vertex $(4, 5)$, that opens down and is congruent to $y = 4x^2$

CONGRUENT: _____



3.7 Equation from Details

Determine the equation of the parabola with vertex $(4, 5)$, that opens down and is congruent to $y = 4x^2$.

Example 3

Determine the equation of the parabola with vertex $(-3,8)$, that opens down, and is congruent to $y = \frac{1}{5}x^2$.

3.8 Equation from Details

Determine the equation of the parabola with vertex $(-4,3)$ that passes through the point $(4,43)$.

3.9 Equation from Details

Example 4

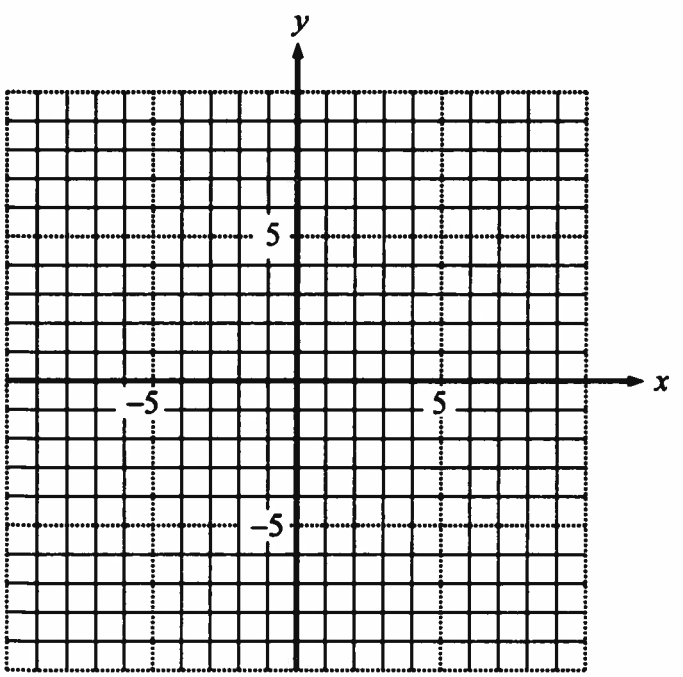
Determine the equation of the parabola with vertex $(-6,-13)$ that passes through the point $(-16,-137)$.

3.10 Equation from Details

Determine the equation of the parabola with vertex $(-6, -21)$, that has a y-intercept of 3.

3.11 Equation from Details

Determine the equation of the parabola that has x-intercepts -3 and -9 with a maximum value of 4.



Lesson 4
Changing from General Form to Standard Form

4.1 General to Standard Form

Simplify: $(x + 7)^2$

Standard Form

$y = (x + 4)^2 - 3$

Simplify:

$y = (x+4)^2 - 3$

_____ same parabola as _____

4.2 General to Standard Form

$y = x^2 - 6x + 16$ What are the coordinates of the vertex?

Skill Building

Simplify: $(x-3)^2 =$

Factor:

Factor: $x^2 + 8x + 16 =$

we don't know what number goes here!

$x^2 - 10x \square = (x \quad)^2$ **Perfect Square**

$x^2 - 6x + 9 = (x-3)(x-3)$

$x^2 + 8x + 16 = (x+4)(x+4)$

Notice how we get the last number $x^2 - 10x \boxed{\quad} = (x \quad)^2$

4.3 General to Standard Form

Example 1 For the following questions find the missing number to complete the square.

1.) $x^2 + 16x$ _____ = _____

3.) $x^2 + 7x$ _____ = _____

2.) $x^2 - 20x$ _____ = _____

4.) $x^2 - \frac{1}{2}x$ _____ = _____

4.4 General to Standard Form

What is the purpose of completing the square?

Write quadratics in the form:

Example

Determine the vertex of $y = x^2 + 16x + 1$

4.5 General to Standard Form

Example 2 Determine the vertex of the following quadratic functions.

1.) $y = x^2 - 14x - 5$

3.) $y = x^2 - 3x + 1$

2.) $y = x^2 + 28x - 13$

4.) $y = x^2 + \frac{2}{3}x + 5$

4.6 General to Standard Form

$y = x^2 + 16x + 1$

$y = x^2 - 16x - 5$

$y = x^2 - 3x + 1$

$y = x^2 + 28x - 13$

$y = x^2 + \frac{2}{3}x + 5$

4.7 General to Standard Form

Determine the vertex of this quadratic function.

$y = 2x^2 + 12x - 3$

4.8 General to Standard Form

Example 3 Determine the vertex of the following quadratic function.

$y = 3x^2 - 15x - 7$

4.9 General to Standard Form

Skill Building

$y = 3x^2 - 15x - 7$

Factor:

Factor:

Factor:

Example 4

Factor the coefficient of the squared term out of both terms for each expression.

a) $4x^2 + 6x$

b) $-5x^2 + \frac{1}{5}x$

c) $\frac{1}{3}x^2 + 4x$

d) $-\frac{3}{4}x^2 - 6x$

4.10 General to Standard Form

Example 5 Determine the coordinates of the vertex for the following quadratic functions.

a) $y = 3x^2 + 8x - 13$

b) $y = -2x^2 + 11x$

Example 6 Determine the maximum or minimum value for the following quadratic functions.

a) $y = 4x^2 + 3x - 5$

b) $y = -\frac{1}{2}x^2 - 5x + 7$

Example 7 Determine the coordinates of the vertex, the equation of the axis of symmetry and The range for the following quadratic functions.

a) $y = -2x^2 + \frac{1}{4}x - 3$

b) $y - 2 = -\frac{1}{3}x - \frac{1}{5}x^2$

4.11 Applications of Quadratic Functions

Example 8

Future astronauts must have the ability to work in zero-gravity conditions. Potential astronauts are trained to work in weightlessness by flying in a modified jet aircraft that is flown in a parabolic path to simulate the weightlessness of space. The function $h(t) = -10t^2 + 300t + 9750$ represents the path of the aircraft, where $h(t)$ is the altitude of the aircraft in metres and t is the time in seconds since weightlessness is achieved.

4.12 Applications of Quadratic Functions

Example 9

The sum of two numbers is 16. Find the numbers if the sum of their squares is a minimum.

4.13 Applications of Quadratic Functions

Example 10

A farmer wants to fence a rectangular area to make a corral by using the bank of a river as one side and then enclosing the other three sides with 160 feet of fence. Find the dimensions of the rectangle that give the maximum area inside.

4.14 Applications of Quadratic Functions

Example 11

An International Zoo charges $8 admission, and averages 2000 visitors per day. A survey shows that, for each $1 increase in the admission cost, 100 fewer people would visit the zoo. What admission cost gives the maximum revenue?

4.15 Applications of Quadratic Functions

Example 12

A bridge is built in the shape of a parabolic arch. The bridge has a span of 192 feet and a maximum height of 30 feet. Find the height of the arch at 40 feet from its center.