

# UNIT 7: ARITHMETIC AND GEOMETRIC SEQUENCES AND SERIES

Note-Taking Supplement

Student Package

Student's Name: \_\_\_\_\_

- ☐ Once completed, submit this package to your Learning Facilitator.
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Lesson 1

Arithmetic Sequences

1.1 Arithmetic Sequences

Definition:

Identify the first term ( $a$ ), and the common difference ( $d$ ) of this arithmetic sequence and then list the next three terms.

8, 12, 16 ...

$\frac{1}{4}, \frac{7}{12}, \frac{11}{12}, \dots$

1.2 Arithmetic Sequences

**Example 1 a)** Given that the first term  $[a]$  is  $-3$  and the common difference is  $[-9]$  in an arithmetic sequence, write out the first four terms.

**b)** Given that the first term  $[a]$  is  $-7$  and the common difference  $[d]$  is  $5$  in an arithmetic sequence, write out the first four terms.

### 1.3 Arithmetic Sequences

If the first term in an arithmetic sequence is represented by the letter [a] and the common difference between terms is represented by the letter [d] we can find an expression for each successive term in the arithmetic sequence in terms of [a] and [ad]

### 1.4 Arithmetic Sequences

53, 48, 43, 38, 33, 28, 23, 18, 13, 8, 3, -2, -7, ...

If  $t_n = 18$ , then what is  $n$  equal to?  $n =$

What is  $t_{12}$  equal to?  $t_{12} =$

## 1.5 Arithmetic Sequences

$$t_n = a + (n - 1)d$$

Given the general term formula for an arithmetic sequence we can use it to:

1)

2)

3)

4)

Given the following is an arithmetic sequence what is the value of term 10?

-4, 1, 6...

## 1.6 Arithmetic Sequences

**Example 2** Given the following is an arithmetic sequence what is the value of term 8?

-22, -13, -4...

## 1.7 Arithmetic Sequences

**Example 3** Given the following is an arithmetic sequence find, an expression for the equation of the general term and then use your equation to find the value of the 100<sup>th</sup> term.

a)  $3, -5, -13 \dots$

b)  $\frac{1}{2}, \frac{1}{4}, 0 \dots$

## 1.8 Arithmetic Sequences

Given the following is an arithmetic sequence, find the number of terms in the sequence.

$44, 40, 36 \dots - 32$

**Example 4** Find the number of terms in the following arithmetic sequence.

$-18, -11, -4 \dots 66$

## 1.9 Arithmetic Means

An arithmetic sequence has six terms. Given the first term in the sequence is 4, and the last term is 19, find the common difference between the terms and then list the missing four terms which are called arithmetic means.

## 1.10 Arithmetic Sequences

### Example 6

a) An arithmetic sequence has four terms. Given the first term in the sequence is  $\frac{3}{4}$  and the

last term is  $\frac{9}{4}$ , find the common distance between the terms and then list the missing

b) An arithmetic sequence has the following terms.

$$-2x - 1, \quad \underline{\hspace{2cm}}, \quad 4x + 7$$

Determine the arithmetic mean and then determine an expression for the common difference.

## 1.11 Finding the First Term

The third term in an arithmetic sequence is  $-2$ , and the eighth term is  $23$ . Find the common difference between the terms and the value of the first term.

The second term in an arithmetic sequence is  $6$ , and the ninth term is  $-50$ . Make up a system of two equations that could be used to solve for the first term and the common difference.



## 1.12 Arithmetic Sequences

### Example 6

In one section of theatre seating there are 6 seats in the row closet to the stage, 7 seats in the row behind, and 8 seats in the next row. How many seats are in the 15<sup>th</sup> row?

## Lesson 2

## Arithmetic Series

## 2.1 Adding Terms by Gauss

Find the sum of the numbers from 1 to 100.

Can you think of another quick method to add these numbers which is very similar to what Gauss did?

## 2.2 Definition of an Arithmetic Series

## 2.3 Finding the Sum of an Arithmetic Series

Find the sum of the first twenty terms in the following arithmetic series.

$$+11 + 15 + 19 \dots$$

## 2.4 Finding the Sum of an Arithmetic Series

### Example 1

a) Find the sum of the first 40 terms in the following arithmetic series.

$$5 + 8 + 11 + \dots$$

b)  $13 + 5 - 3 - 11 - \dots S_{26}$

## 2.5 Finding the Sum of an Arithmetic Series

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

**Example 2** Determine the sum of the following arithmetic series.

$$11 + 17 + 23 + \dots + 215$$

## 2.6 Finding the Sum of an Arithmetic Series

**Example 3**

a) Find the sum of the following arithmetic series using the following formula.  $S_n = \frac{n}{2}[2a + (n-1)d]$

b) Find the sum of the following arithmetic series using the following formula.

b) Find the sum of the following arithmetic series using the following formula.  $S_n = \frac{n}{2}[a + l]$

$$\frac{2}{5} + \frac{4}{5} + \frac{6}{5} + \dots + \frac{56}{5}$$

## 2.7 Arithmetic Series

**Example 4** Find an expression for the sum of the following arithmetic series.

$$(2x - 3y) + (3x - 2y) + (4x - y) + \dots + (10x + 5y)$$

## 2.8 Finding Terms of an Arithmetic Series

### Example 5

The sum of the first four terms of an arithmetic series is 36 and the sum of the first ten terms is 210.

Determine the first ten terms of the series and the sum to twenty terms.

## 2.9 Finding Terms of an Arithmetic Series

### Example 6

The sum of the first four terms of an arithmetic series is 48 and the sum of the first six terms is 24.

Determine the first six terms of the series and the sum to fifteen terms.

**2.10 Application of Arithmetic Series**

**Example 7**

George started working for a bank. His starting salary was \$50 000 and in his fifth year he made \$60 000. His salary increases formed an arithmetic sequence. How much was his yearly raise? In what year would he make \$80 000? If he worked at the same job for 25 years, what would his total income be over 25 years?

## Lesson 3

## Geometric Series

## 3.1 Geometric Sequences

Sequence 1 4, 11, 18, 25, 32 ...

Sequence 2 2, 16, 18, 54, 162 ...

What are the next terms in each sequence?

The following are geometric sequences. Determine the common ratio for each.

3, 15, 75, 375 ...

18, 9, 4.5, 2.25 ...

2, -8, 32, -128 ...

 $a, ar, ar^2, ar^3 \dots$ 

Are all sequences Geometric?

Example 1 Is each sequences Geometric?

a) 2, 4, 6, 10, 16 ...

b) 4, -2, 1,  $-\frac{1}{2}$ ,  $\frac{1}{4}$  ...

c) 0.5, 0.55, 0.555, 0.5555..

d) 0.7, 0.07, 0.007, 0.0007...



## 3.2 Geometric Sequences

$-8, -4, -2, -1, \dots$

$-8, -4, -2, -1, \dots$

## 3.3 The General Term of a Geometric Sequences

$3, 6, 12, 24, 48 \dots$

**Example 2** What is the 30<sup>th</sup> term of the geometric sequence?

### 3.4 The General Term of a Geometric Sequences

$$t_1 =$$

**Example 3** For the geometric sequence, determine the general term ( $t_n$ ). Then use it to determine

the 10<sup>th</sup> term  $t_{10}$

8, -4, 2, -1, ...

### 3.5 Finding The General Term

**Example 4** In a geometric sequence 3, 12, 48, ... determine:

a)  $t_n$

b)  $t_{13}$

### 3.6 Review Exponential Equations

Write the following number as a power of 7.

49

What value of  $x$  makes this equation equal?

$$6^x = 6^3$$

$$4^x = 6^3$$

$$7^5 = 7^{n-1} \quad \text{Solve for } n.$$

Example 5  $32 = 2^{n-1}$  Solve for  $n$ .

Example 6 Solve the following equations.

a)  $4096 = 4^{n-1}$

b)  $3^{2n-1} = 19683$

### 3.7 Using the General Term

**Example 7** In a geometric sequence 5, 10, 20, ... determine:

a) the 11<sup>th</sup> term

b) which term is 2 621 440?

### 3.8 Using the General Term

**Example 8** In the geometric sequence  $t_5 = 48$ , and  $t_8 = 384$ . Find the first two terms.

### 3.9 Using the General Term

#### Example 9

In a geometric sequence, the 3<sup>rd</sup> term is 20 and the 6<sup>th</sup> term is -540. Find the first six terms.

### 3.10 Problems Involving Geometric Sequences

#### Example 10

A rubber ball is on a shelf 4 m high. The shelf is bumped and the ball begins to roll and falls to the floor.

After each bounce it rises to a height that is 73% of the previous height.



a) Write the general term of the sequence that relates the height of the bounce to the number of bounces.

## b) What height does the ball reach after its 9<sup>th</sup> bounce?

## 3.11 Problems Involving Geometric Sequences

### Example 11

An aquarium originally containing 50 litres of water loses 6% of its water to evaporation every day

Determine the amount of water left in the aquarium at the end of the 11<sup>th</sup> day.

**Lesson 4**  
**Geometric Series****4.1 Geometric Series****Geometric Sequence –****Geometric Series –****4.2 Geometric Series****Example 1 Determine the sum of the first 13 terms of each geometric series.**

**a)  $4 + 8 + 16 + 32 + \dots$**

**b)  $54 + 18 + 6 + 2 + \dots$**

$$\text{c) } -\frac{1}{3} + \frac{4}{9} - \frac{16}{27} + \frac{64}{81} + \dots$$

## 4.3 Geometric Series

**Example 2** Determine the sum of this geometric series with an unspecified number of terms.

$$4 - 12 + 36 - \dots - 8748$$

## 4.4 Geometric Series

**Example 3** Determine the sum of this geometric series with an unspecified number of terms.

$$4 - 12 + 36 - \dots - 8748$$



## 4.5 Geometric Series

**Example 4** Determine the sum of following geometric series.

a)  $5 + 20 + 80 + \dots + 20480$

b)  $32 + 16 + 8 + \dots + \frac{1}{8}$

## 4.6 Geometric Series

**Example 5** The sum of the first 15 terms of a geometric series is 131 068 and the common ratio is 2. Determine the value of the first term.

## 4.7 Geometric Series

**Example 6** An oil well The North American Chess Championships has attracted 131 072 players.

**When a player loses a match, the player is eliminated from the tournament and the winner moves on to the next round. How many matches are played until the champion is declared.**

## 4.8 Geometric Series

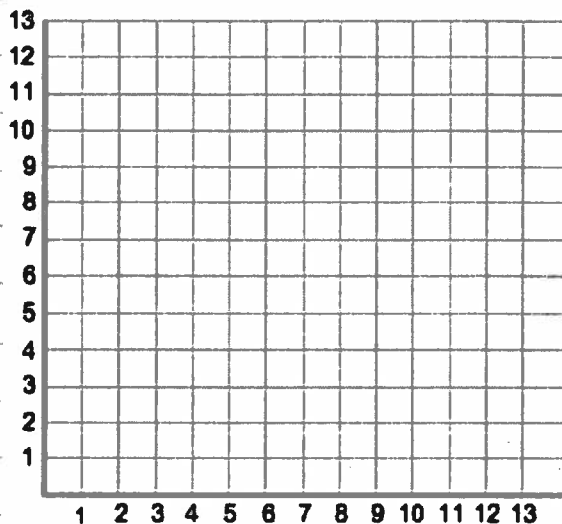
**Example 7** An oil well produces 22 000 barrels of oil in its first month of production. The wells production drops by 2.1% per month. What is the total amount of oil produced by this well in the first 3 years of production?

**Lesson 5**  
**Infinite Geometric Series**

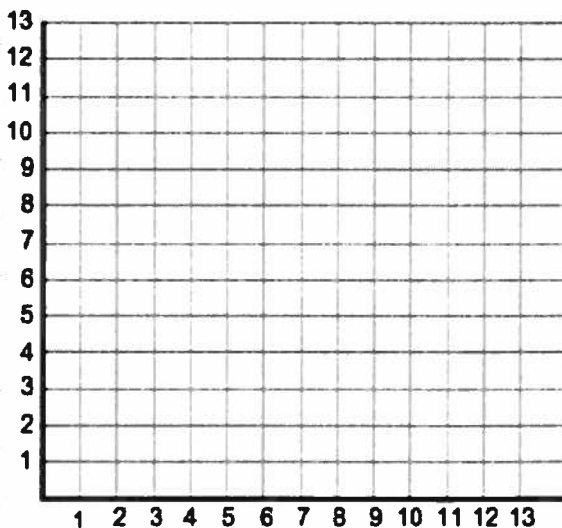
**5.1 Infinite Geometric Series**

Find the sum of an infinite Geometric Series.

$$2 + 6 + 18 + 54 + 162 + \dots$$

**5.2 Infinite Geometric Series**

$$3 + 1.5 + .75 + .375 + \dots$$



### 5.3 Infinite Geometric Series

$$2 + 6 + 18 + 54 + 162 + \dots$$

$$3 + 1.5 + .75 + .375 + \dots$$

How do you determine if a Geometric Series has No Sum or a SUM?

**Example 1** Determine which infinite geometric series has sum.

a)  $100 + 50 + 25 + 12.5 + \dots$     b)  $5 - 15 + 45 - 135 + \dots$     c)  $0.1 + 0.01 + 0.001 + 0.0001 + \dots$

a)  $2 + 8 + 32 + 128 + \dots$     b)  $-16 + 12 - 9 + 6.75 + \dots$     c)  $8 - 12 + 18 - 27 + \dots$

### 5.4 Finding the Sum of Infinite Geometric Series

$$3 + 1.5 + .75 + .375 + \dots$$

**Example 2 Find the common ratio for each of the following geometric series and state whether a sum To infinity exists. Find the sum where it exists.**

a)  $10 - 9 + 8.1 - \dots$

b)  $.0001 + .001 + .01 + \dots$

c)  $7 + \frac{7}{3} + \frac{7}{9} + \dots$

d)  $\frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \dots$

## 5.5 Problems Involving Infinite Geometric Series

**Example 3 The sum of an infinite geometric series is 27 and the first term is 18. Find the common ratio.**

**Example 4 An employee at Environment Canada releases a weather balloon. The balloon rises 200 m in the first minute. Each minute there after it rises 5% less than the previous minute. What is the maximum height reached by the balloon?**

**Example 5 An oil well produces 30 000 barrels of oil in the first month of production. The wells production drops 4.1% per month. Estimate the total number of barrels in the well.**

## 5.6 Problems Involving Infinite Geometric Series

### Example 6

A rubber ball is on a shelf 5 m high. The shelf is bumped and the ball begins to roll and falls to the floor. After each bounce it rises to a height that is 70% of the previous height.



a) What is the total vertical distance the ball has travelled when it strikes the ground the eleventh time?

b) Estimate the total vertical distance the ball travels before coming to rest.