

UNIT 8: TRIGONOMETRY

Note-Taking Supplement

Student Package

Student's Name: _____

- ☐ Once completed, submit this package to your Learning Facilitator.
- ☐ Click on the "Unit 8 Note-Taking Supplement" link in your Moodle course and follow the instructions to submit your request for marking.

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Mark (out of 20):

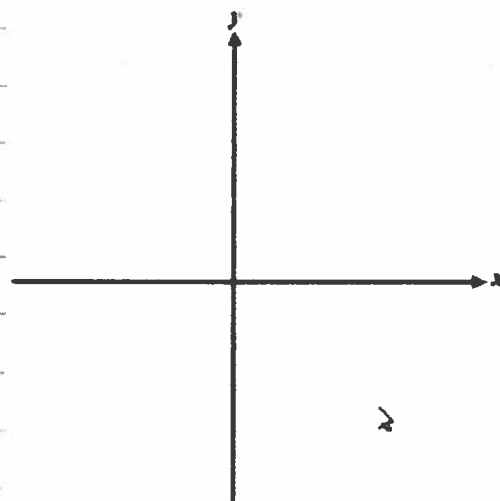
Lesson 1

Angles in Standard Position

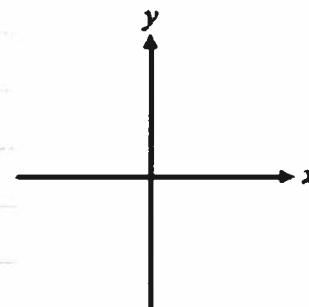
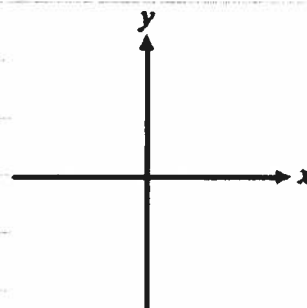
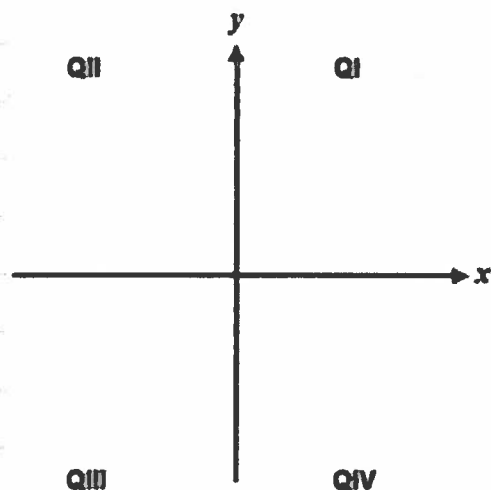
1.1 Angles in Standard Position

GEOMETRY

TRIGONOMETRY

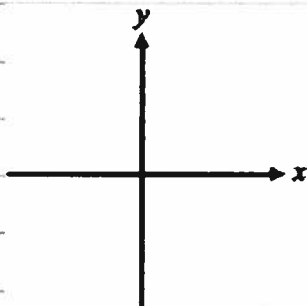


1.2 Angles in Standard Position



1.3 Angles in Standard Position

Draw 125° in standard position.



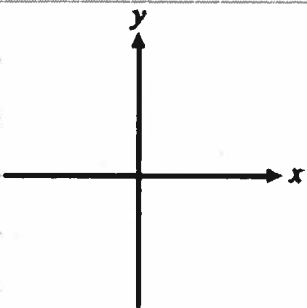
Example 1 Draw each angle in standard position

a) 250°

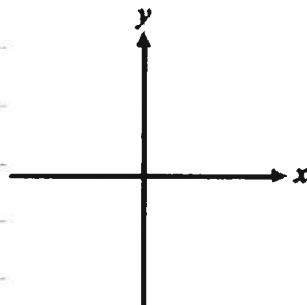
b) 315°

1.4 Reference Angle

Draw 125° in standard position.



Draw 48° in standard position.



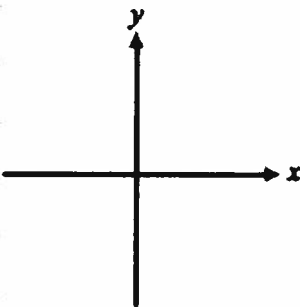
1.5 Reference Angle

Example 2 Determine the measure of the reference angle for each angle in standard position.

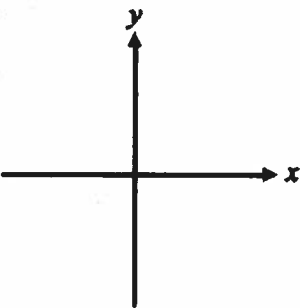
a) 230°

b) 300°

1.6 Same Reference Angle

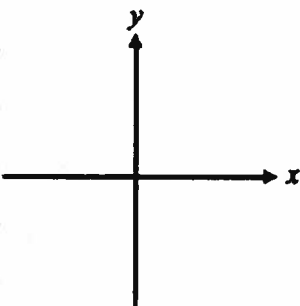


What is the measure of the angle in standard position if you reflect the 60° angle in the y -axis?



What is the measure of the angle in standard position if you reflect the 60° angle in the x -axis?

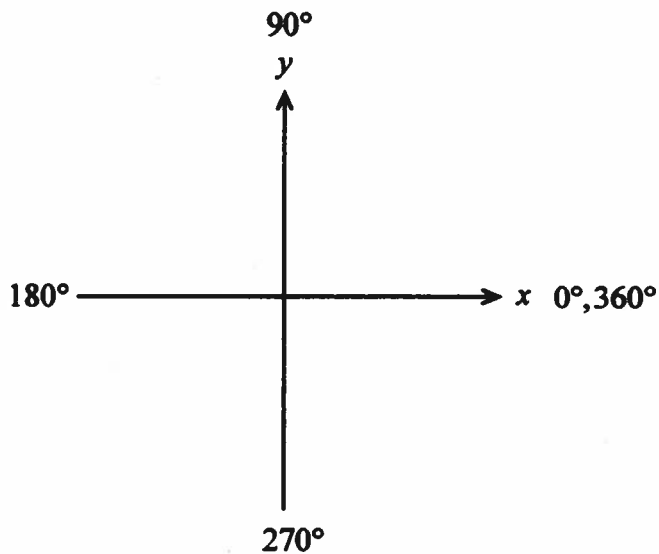
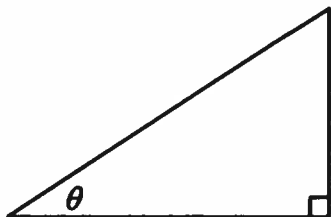
There is one other angle between 0 degrees and 360 degrees that has the same reference angle.



1.7 Same Reference Angles

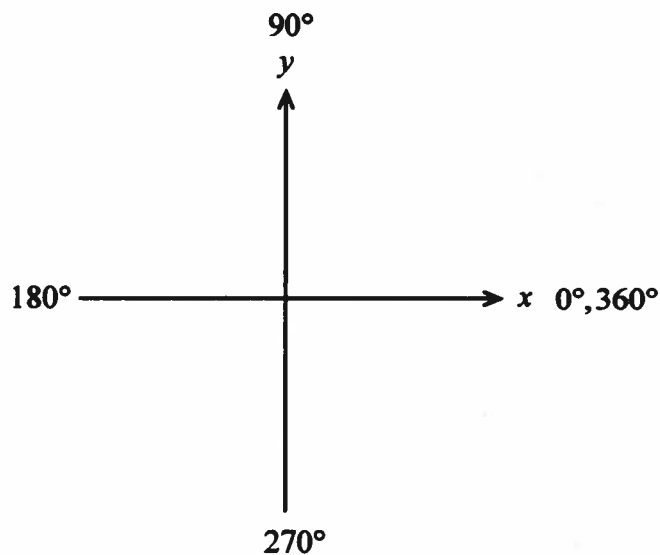
Example 3 An If 52° is in standard position, then determine the measure of three other angles in Standard position with the same reference angle where $0^\circ < \theta < 360^\circ$

Lesson 2
Trig Ratios of Angles in Standard Position

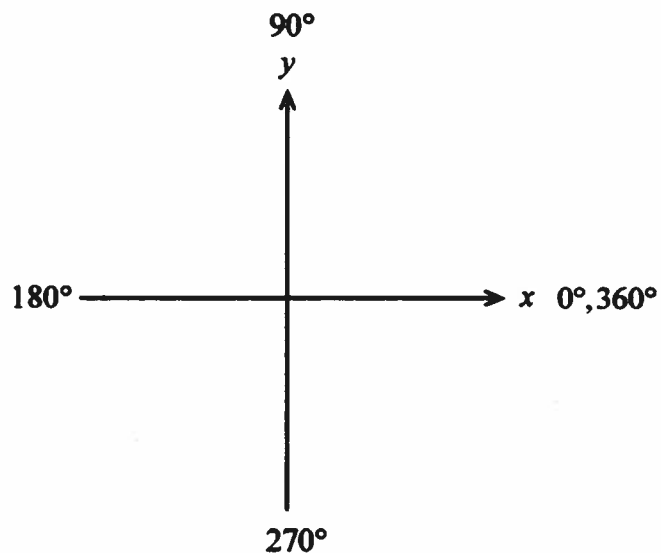
2.1 Primary Trig Ratios**2.2 Trig Ratios of Angles in Standard Position**

$$\cos 53^\circ = \underline{\hspace{2cm}}$$

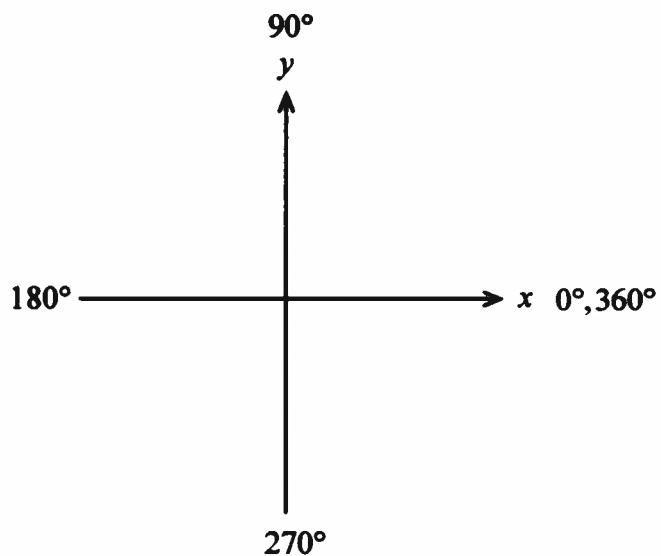
$$\cos 127^\circ = \underline{\hspace{2cm}}$$



2.3 Trig Ratios of Angles in Standard Position



What angle in Standard Position in QIV has the same digits as Cosine 214 degrees?



$$\tan 320^\circ = \underline{\hspace{2cm}}$$

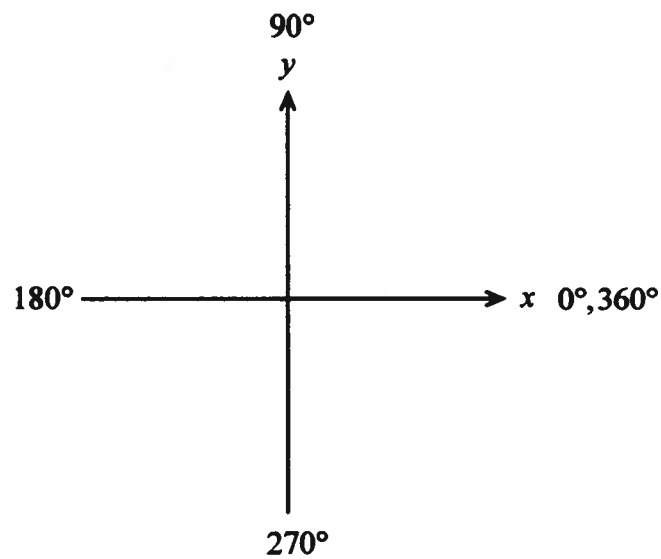
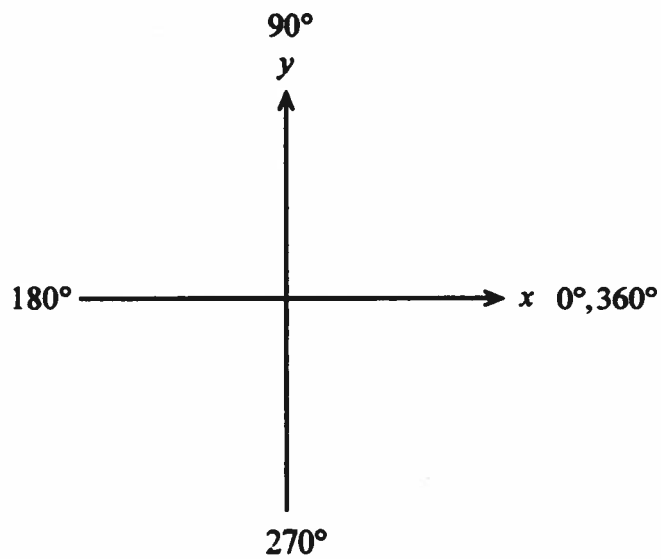
Determine the tangent of 320° reference angle.

$$\sin 255^\circ = \underline{\hspace{2cm}}$$

$$\sin(\text{ref angle}) = \underline{\hspace{2cm}}$$

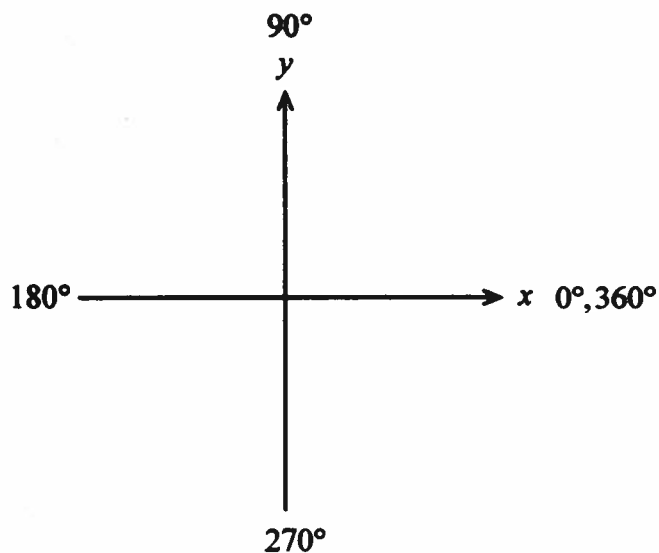
2.4 Trig Ratios of Angles in Standard Position

$P(x, y)$ on the terminal arm



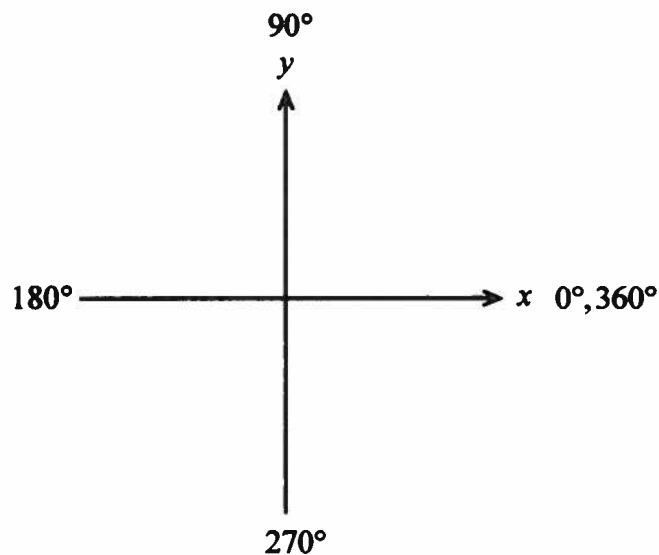
2.5 Trig Ratios of Angles in Standard Position

Example 1 Point $P(-6, -5)$ is on the terminal arm of angle θ in Standard Position.
Calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$.

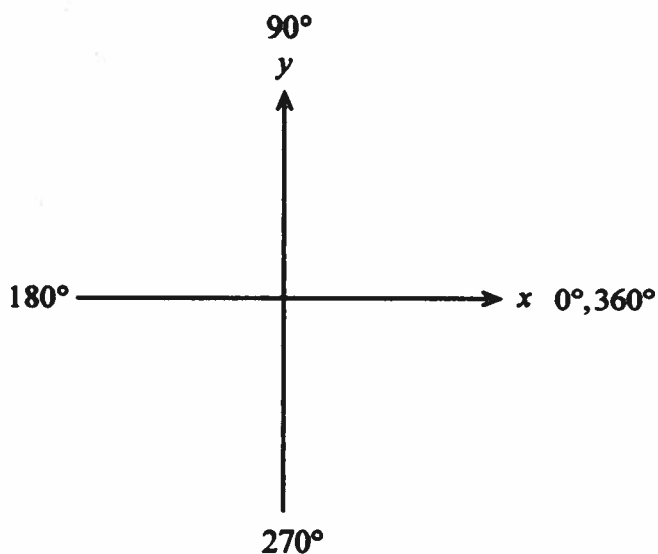


2.6 Trig Ratios of Angles in Standard Position

Example 2 Point $P(4, -7)$ is on the terminal arm of angle θ in Standard Position.
Calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$.

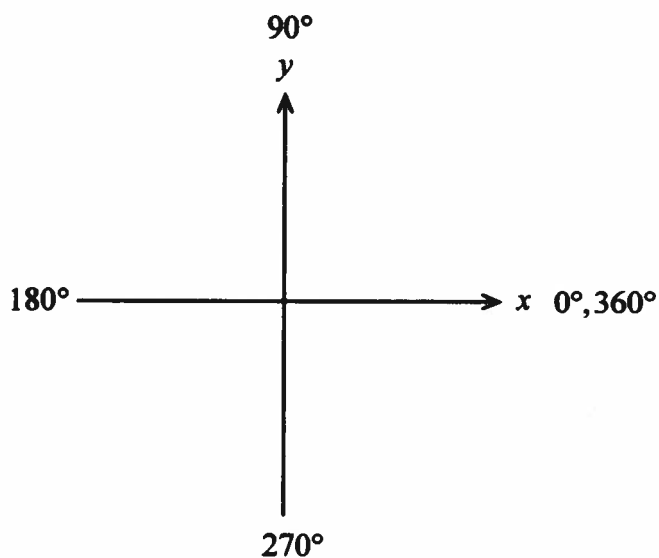


2.7 Trig Ratios of Angles in Standard Position



2.8 Trig Ratios of Angles in Standard Position

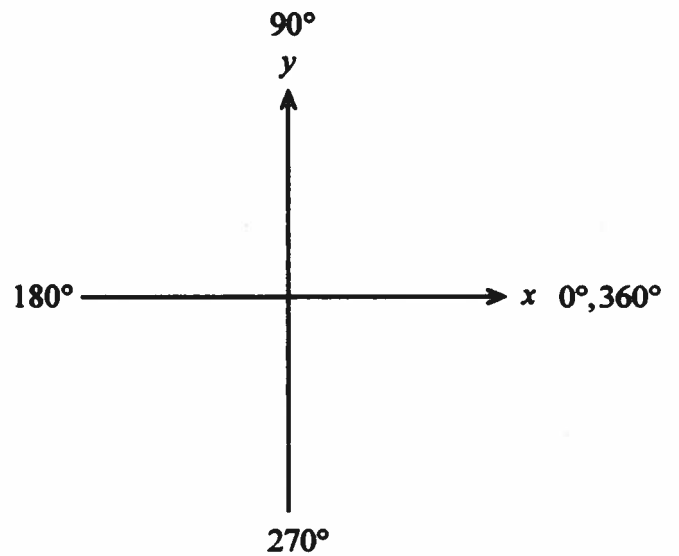
Example 3 Angle θ lies in quadrant IV and $\sin \theta = -\frac{7}{11}$
Find the exact values of the other two trigonometric ratios.



2.9 Trig Ratios of Angles in Standard Position

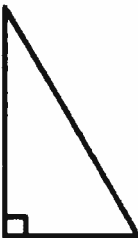
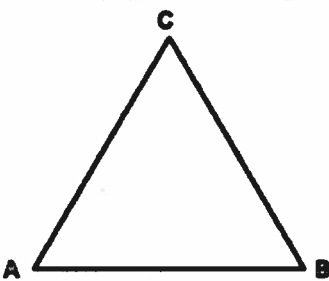
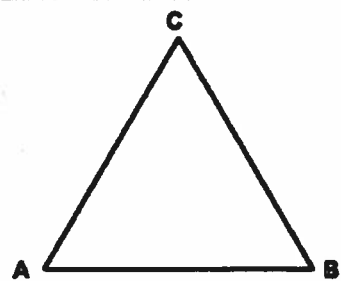
Example 4 Angle θ lies in quadrant II and $\sin \theta = \frac{4}{9}$

Find the exact values of the other two trigonometric ratios.



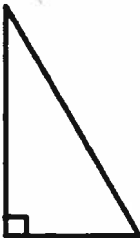
Lesson 3
Exact Values Sine, Cosine, and Tangent of Special Angles

3.1 30-60-90 Triangle

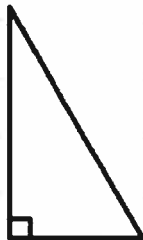


Find the EXACT length of CD using
Pythagorean Theorem.

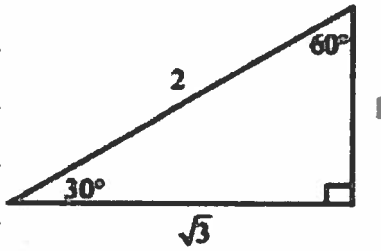
30° – 60° – 90° Triangle



3.2 30-60-90 Triangle



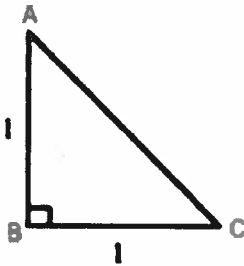
Decimal approximation:



EXACT VALUE

3.3 45-45-90 Triangle

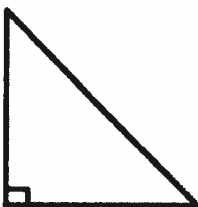
Isosceles right triangle.

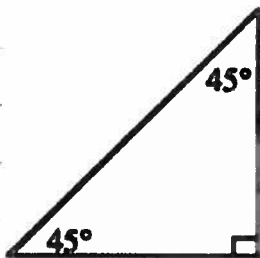


What is the measure of $\angle A$ and $\angle C$?

What is the exact length of AC?

45°–45°–90° Triangle

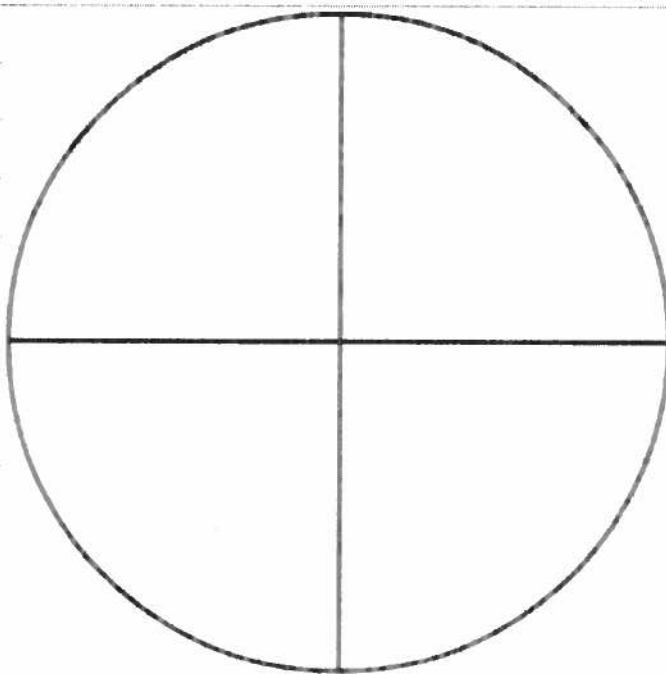




EXACT VALUE

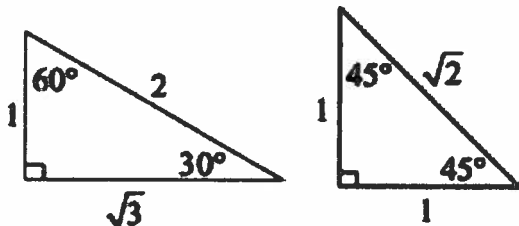
3.4 The Trig Circle

**Exact Value of : sine, cosine and tangent of
 30° , 45° and 60° .**



3.5 Special Angles in the Trig Circle

Special Triangles



$$\sin = \frac{\text{opp}}{\text{hyp}}$$

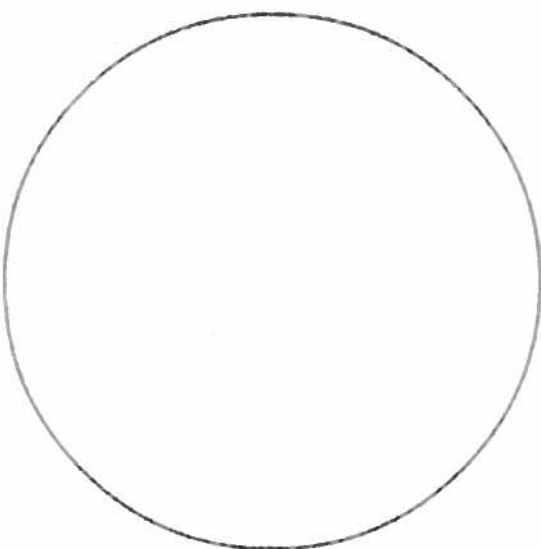
$$\cos = \frac{\text{adj}}{\text{hyp}}$$

$$\tan = \frac{\text{opp}}{\text{adj}}$$

	30°	45°	60°
sin			
cos			
tan			

Example 1 Determine the EXACT value of $\cos 150^\circ$.

The Trig Circle



3.6 Exact Values of Special Angles

Example 2

Determine the EXACT value of $\sin 225^\circ$.

Lesson 4

Solving Conditional Trigonometric Equations

4.1 Conditions on θ

$$\sin 35^\circ = ?$$

$$\sin \theta = .89$$

$$\sin 35^\circ =$$

$$\theta =$$

4.2 Conditions on θ

If $\sin \theta = .64$, determine θ .

$$\sin 40^\circ =$$

$$\sin 400^\circ =$$

$$\sin 760^\circ =$$

$$\sin 1120^\circ =$$

Place conditions on θ .

Solve the equation $\sin \theta = .72$ for θ to the nearest degree, where $0^\circ \leq \theta < 360^\circ$.

4.3 Conditions on θ

Example 1 Find each angle to 2 decimal places if $0^\circ \leq \theta < 360^\circ$ for each of the following.

a) $\cos \theta = -.386$

b) $\tan \theta = 5.72$

4.4 Solving Conditional Trig Equations

Example 2 Solve the equation $7\sin \theta + 5 = 0$ for θ , where $0^\circ \leq \theta < 360^\circ$.

4.6 Solving Conditional Trlg Equations

Example 3 Solve the equation $4 \tan \theta - 11 = 0$ for θ to 2 decimal places, where $0^\circ \leq \theta < 360^\circ$.

4.6 Solving Conditional Trlg Equations

Example 4 Solve the equation $5 \cos^2 \theta - 3 = 0$ for θ to 2 decimal places, where $0^\circ \leq \theta < 360^\circ$.

4.7 Solving Conditional Trig Equations

Example 5 Solve the equation $6\sin^2\theta + \sin\theta - 1 = 0$ for θ to 2 decimal places, where $0^\circ \leq \theta < 360^\circ$.

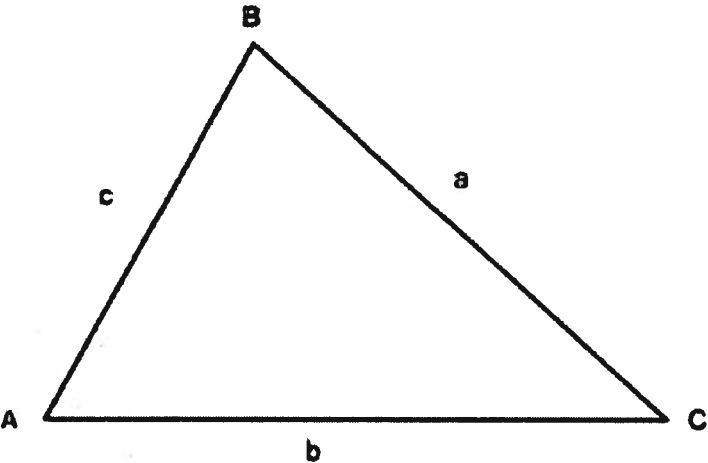
4.8 Solving Conditional Trig Equations

Example 5 For the equation $2\cos\theta + \sqrt{3} = 0$ find the EXACT value of θ , where $0^\circ \leq \theta < 360^\circ$.

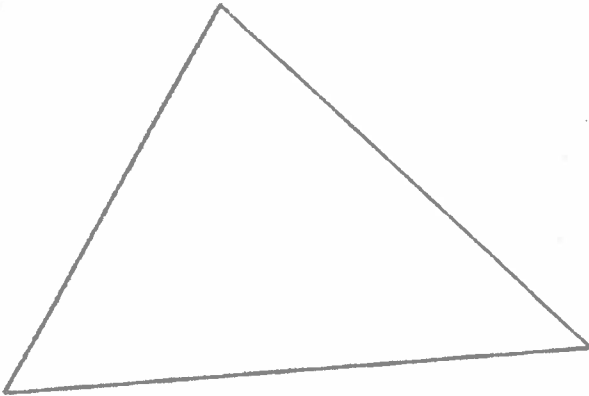
Lesson 5
The Sine Law

5.1 Creating the Sine Law

The primary trigonometric ratios can be applied to an acute triangle.



5.2 The Sine Law Formula

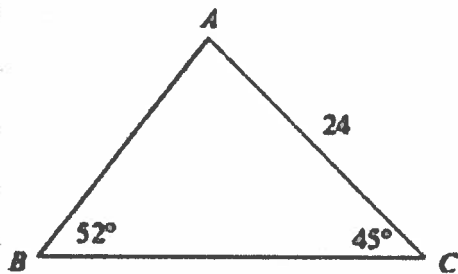


5.3 When to Apply the Sine Law

How do we know when to use the Sine Law?

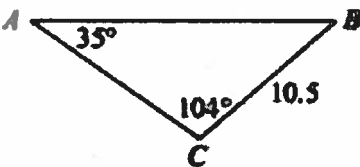
5.4 Using the Sine Law

Example 1 Find the length of AB to the nearest tenth.



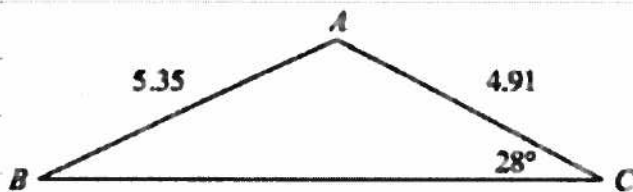
5.5 Using the Sine Law

Example 2 Find the length of AB rounded to 1 decimal place.



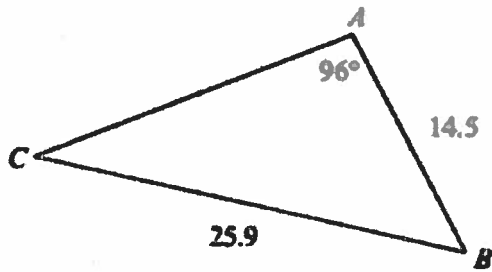
5.6 The Sine Law and ASS Triangle

Example 3 Find the angle at B to the nearest degree.



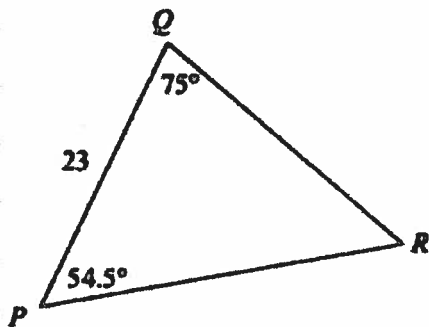
5.7 Sine Law and ASS Triangle

Example 4 Find the angle at C to the nearest degree.



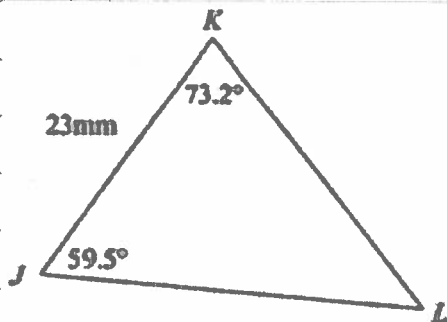
5.8 Using the Sine Law

Example 5 Find the length of the side PR rounded to the nearest tenth.



5.9 The Sine Law and ASA Triangle

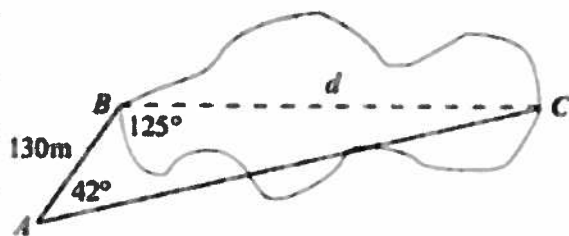
Example 6 Find the length of the side KL to the nearest tenth.



5.10 Practical Application of the Sine Law

Example 7 To measure the length of a lake, a baseline AB is established and measured to be 130 m.

Angles A and B are measured to be 42° and 125° respectively. How long is the lake?



5.11 Practical Application of the Sine Law

Example 8 A ship is heading due east and passes rock A. At this time, the bearing to a lighthouse is $N 60^\circ E$. After travelling 5 km, the bearing is $N 40^\circ E$. How far is the ship from the lighthouse?

5.12 Practical Application of the Sine Law

Example 9 An airplane is flying due west and passes overhead of Vancouver. At this time, the bearing to Mount alpha is $S 55^\circ W$. After travelling 100 km the bearing is $S 30^\circ W$. How far is the airplane from Mount Alpha?

5.13 Practical Application of the Sine Law

Example 10

Tom, who stands 6 feet tall, wants to approximate the height of the Washington Monument in Washington DC, USA. Because of a recent earthquake, he is standing at the edge of the restricted area and measures the angle of elevation to the top of the monument to be 56° . He measures a distance of 100 feet farther away from the monument and measures the angle of elevation to be 48° . Determine the height of the Washington Monument to the nearest foot.

5.13 Practical Application of the Sine Law

Example 11

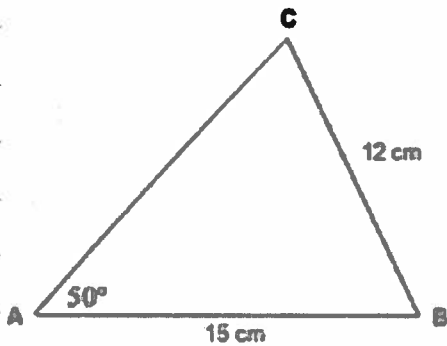
A hot air balloon is flying directly between two cities that are 4 km apart. The balloonist finds that the angle of depression to one city is 38° and 33° to the other city. How high above the ground is the balloon?

Lesson 6

Ambiguous Case of The Sine Law

6.1 Sine Law and the Ambiguous Case

Given the following triangle, would you use the sine law or the cosine law to find the measure of the unknown angles and the length of the unknown



6.2 Drawing Triangles and the Sine Law

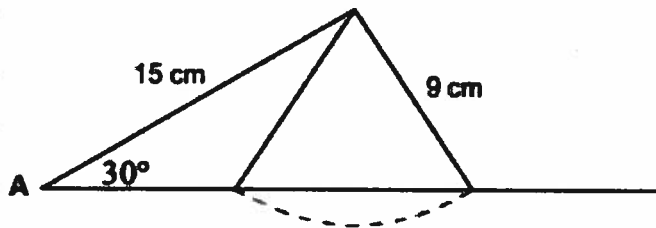
Draw $\triangle ABC$ with the following measurements. $\angle A = 30^\circ$, $a = 9$ cm, $b = 15$ cm

Aleah's Drawing

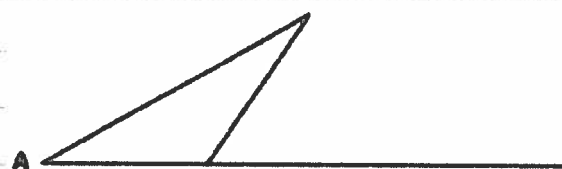
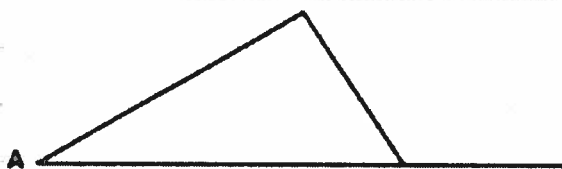
Michaela's Drawing

6.3 Drawing Triangles and the Sine Law

How do we know when to use the Sine Law?



Unique Feature of Ambiguous Case



6.4 Drawing Triangles and the Sine Law

Draw $\triangle ABC$ with the following measurements. $\angle A = 30^\circ$, $a = 9$ cm, $b = 15$ cm

6.5 How Many Triangles

Draw $\triangle DEF$ with the following measurements. $\angle F = 30^\circ$, $f = 7$ cm, $e = 17$ cm

6.6 How Many Triangles

Draw $\triangle JKL$ with the following measurements. $\angle K = 30^\circ$, $k = 9$ cm, $l = 18$ cm

6.7 How Many Triangles

Draw $\triangle MNO$ with the following measurements. $\angle M = 30^\circ$, $m = 21$ cm, $n = 15$ cm

6.8 Sine Law and the Ambiguous Case

Example 1 Solve for all unknown angles and sides in triangle $\triangle DEF$ if:

$$\angle E = 32^\circ, f = 15 \text{ cm}, e = 15 \text{ cm}$$

6.9 Sine Law and the Ambiguous Case

Example 2

Solve for all unknown angle measures and side lengths for triangle ABC given angle A measures 29 degrees, side "a" is 11 metres long, and side "b" is 20 metres long.

Answer degrees and sides rounded to 1 decimal place.

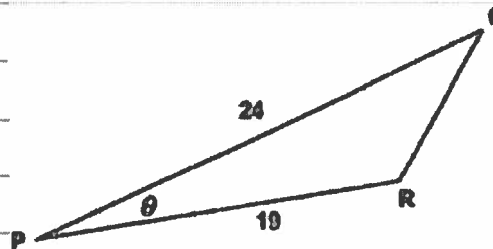
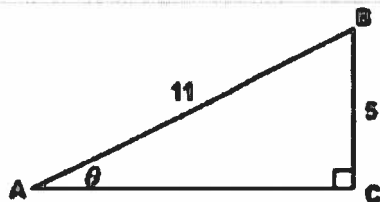
6.10 Sine Law and the Ambiguous Case

Example 3 Solve the following triangle ABC if angle A is 55 degrees, side b is 25 metres long, and side a is 18 metres long.

Lesson 7 The Cosine Law

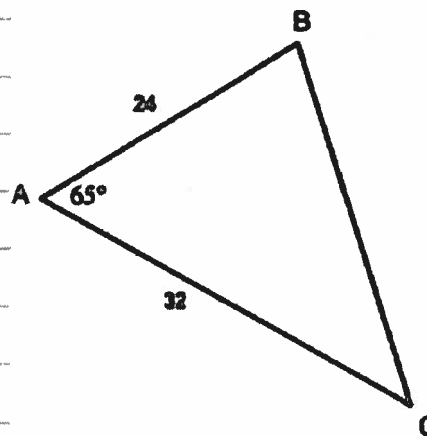
7.1 Right Angled Trigonometry

Primary Trigonometric Ratios



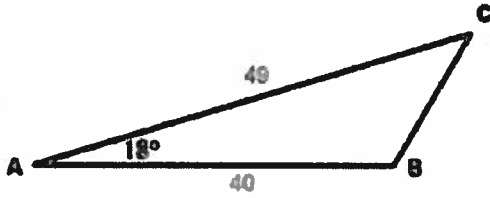
7.2 The Cosine Law

The Cosine Law:



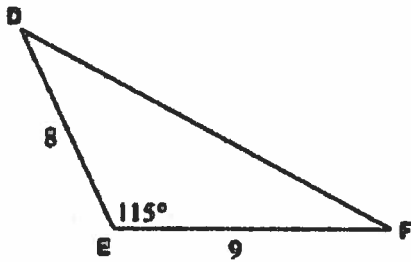
7.3 Cosine Law

Use the Cosine Law to find the length of side "a" to the nearest 10th.

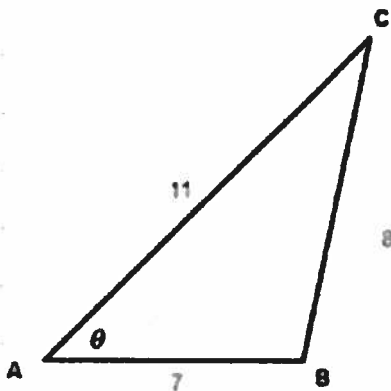


7.4 The Cosine Law

Find the third side of the given triangle.

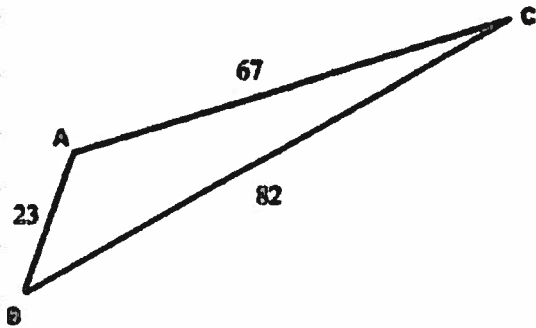


7.5 Cosine Law – Finding an Angle



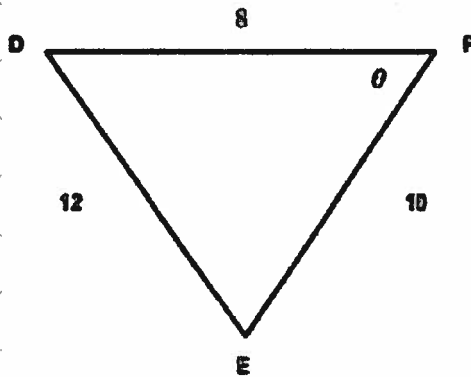
7.6 Cosine Law

Use the Cosine Law to find the measure of angle B to the nearest 10^{th} .



7.7 The Cosine Law

Example 3 Find the missing angle, θ , accurate to one decimal place.



7.8 How Many Triangles

Example 4 For a triangle with sides of given lengths, find to 1 decimal place the measure of the smallest angle.

a) 8, 13, 17

7.9 The Cosine Law

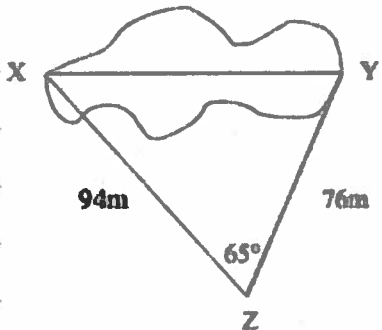
Example 5 For a triangle with sides of given lengths, find to 1 decimal place the measure of the largest angle.

a) 16, 125, 36

7.10 Applying the Cosine Law

Example 6

A surveyor has measured the following distances XZ and YZ, and the $\angle Z$ at 65 degrees. How far is it across the pond? Round answer to 1 decimal place.



7.11 Applying the Cosine Law

Example 7

A Nebo SVU counter-stealth radar has detected an F-35 fighter jet at a range of 41 km and is tracking an Su-35S Russian fighter jet at 34 km. The angle between the two aircraft is 112. How far apart are they rounded to nearest tenth?

7.12 Applying the Cosine Law

Example 8

The goal posts for a hockey net are exactly 6 feet apart, or approximately 1.82 metres. The puck is shot from a point where the distance to one post is 9.4m and the distance to the other is 10.2m. From where the puck is shot, what is the angle between the posts to the nearest degree?

7.13 Applying the Cosine Law

Example 9

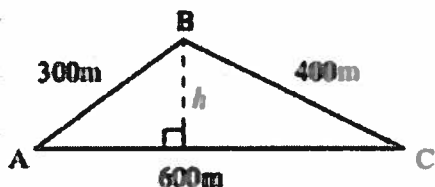
A course for a sailing race follows a triangular route around three buoys. The distances between the three buoys are 9.12 km, 7.46 km, and 10.31 km. What is the angle at the buoy which is adjacent to the 7.46 km and 9.12 km sides?

7.14 Applying the Cosine Law

Example 10

The length of the sides of a triangular parcel of land are approximately 300m, 400m, and 600m.

Approximate the area of the parcel of land to the nearest square metre.



7.15 Applying the Cosine Law

Example 11

The distance from home plate to center field at Yankee Stadium is 400 feet. What is the angle A between shortstop (halfway between second and third base) and home plate?

